ENTRY DETERRENCE BY NON-HORIZONTAL MERGER*

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Abstract
We study when and how pure non-horizontal mergers, whether cross-product or vertical, can deter new entry. Organizational mergers implicitly commit firms to more aggressive price competition. Because heightened competition deters entry, mergers can occur in equilibrium even when, absent entry considerations, they do not. We show that, in order to prevent a flood of entrants, mergers arise even when a marginal merger costs incumbent firms more than does a marginal entrant.

*I owe special thanks to Yeon-Koo Che and two anonymous reviewers for meticulous and insightful comments on earlier versions of this paper. I also want to thank Larry Karp and Steve Hamilton for vibrant discussions that helped stimulate this research. The usual disclaimer applies.

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I. INTRODUCTION

This paper studies pure non-horizontal mergers, whether vertical or cross-product. Vertical mergers combine input and output producers in a single firm. Cross-product mergers combine sellers of different products in a single firm -- a "superstore" vs. a mall, for example. Unlike horizontal mergers, neither of these combinations integrates producers of the same output good. The question we ask is: Do such non-horizontal mergers deter the entry of rival producer / retailer chains? And if so, do such entry-deterrence considerations help explain when and why such mergers occur?

Multi-product retailing is pervasive in contemporary economies, whether in supermarkets, "big box" superstores, malls or bundled products (e.g., computer hardware and software). Because "shopping" -- the process of searching for desired products and purchasing them -- is a time-consuming activity, there are economies of retailing many products at one location, whether in physical, product or cyber space (Dudey [1990]). A key question for positive economics is: how might one expect this multi-product retailing to be organized -- for example, in a mall of independent outlets or an integrated (merged) "big box" store? Overall, the literature has given relatively little attention to this question. However, Beggs [1994] offers one possible thesis for the emergence of "malls." By spurring marginal consumers to shop elsewhere, a higher price of one outlet's product can reduce demand at another store; merged outlets, accounting for this cross-store externality, charge lower prices (ceteris paribus).1 Although this practice is profit-enhancing given the pricing regimes of rival retailers, it can prompt rivals to lower their prices as well. In view of this effect, a multi-product retailer can effectively pre-commit to higher prices by organizing itself as a mall of independent outlets. Because rivals respond with higher prices, this pre-commitment can be advantageous.

In this paper, we take this logic a step further by considering the role of mergers in altering entry incentives. Precisely because mergers lead to lower market prices -- their disadvantage in a model without any prospective entry -- they discourage entry of new rivals.
When entry is costly to extant retailers, mergers can thus be advantageous for retail groups, essentially because their anti-competitive effect (limiting entry) can dominate their pro-competitive effect (lowering prices for a given set of competitors). Moreover, this can be true even when mergers are "very costly" to retailers -- indeed, when marginal mergers are more costly than is marginal entry. The reason is that mergers are then needed not only to deter marginal entry, but to prevent an "opening of the floodgates" that prompts the operation of many more retailers than would operate if mergers were made. In Fudenberg and Tirole’s [1984] terminology, merging (vs. not merging) represents a “Top Dog” (vs. “Puppy Dog”) strategy designed to deter (vs. accommodate) entry.

Similar logic applies in the case of vertical mergers, a topic much more exhaustively treated in the literature. A well-known advantage of vertical separation in differentiated product markets (Bonnano and Vickers [1988]) is that separated output producers and input suppliers can sign contracts that precommit them to above-cost wholesale prices, which in turn prompt the output producers to charge higher output prices (ceteris paribus). Because rival output prices are strategic complements, rivals respond to the contractual precommitment by charging higher prices themselves -- the strategic benefit of vertical separation. Here, this logic implies that vertical separation, by prompting higher equilibrium prices, will encourage entry. Conversely, vertical integration -- by voiding the opportunity for contractual precommitments -- deters entry.\(^2\)

This entry-deterrence motive for vertical integration differs from the recent literature on "vertical foreclosure," wherein vertical integration also has the effect of limiting downstream competition. The dominant theme in this work is that vertically integrated firms can exploit their control of an input supplier to raise input prices to which competitors are subject (see, for example, Salinger [1988]; Ordover, Saloner and Salop [1990]; Hart and Tirole [1990]; Bolton and Whinston [1991]; Reiffen [1992]; Riordan [1998]; Economides [1998]; Chen [2001]; Chemla [2003]).\(^3\) A necessary ingredient to these "raising rivals' costs" arguments (Salop and Scheffman [1987]) is that the input market not be perfectly contestable. In this
paper, we void such motives for vertical integration by assuming that input markets are perfectly contestable (with inputs produced at a common constant marginal cost in a free-entry industry).

Our arguments are also closely related to recent work on commodity bundling (notably, Nalebuff [2004a]). In the bundling literature, an incumbent firm has a monopoly in one product, the purchase of which is tied to (or bundled with) another product. There are two ways in which such bundling deters entry into the tied product market. First, bundling forecloses much of the market for a potential entrant (e.g., see Whinston [1990]; Choi and Stefanadis [2001]; Carlton and Waldman [2002]). Second, because “bundled consumers” are more valuable than “individual product consumers,” a bundling monopoly has a greater incentive to attract customers by cutting price, which also deters entry (Nalebuff [2004a]).

In the present paper, we are not concerned with monopoly bundling per se, rather with non-horizontal mergers in oligopolistic markets. Like Nalebuff’s [2004a] commodity bundles, mergers provide incentives to cut prices and these incentives deter entry. However, there is a key difference in oligopoly (vs. monopoly) markets. The price-cutting effects of commodity bundles are advantageous to an incumbent monopoly regardless of entry. Here, in contrast, the price-cutting effects of mergers are generally disadvantageous to incumbent oligopolies, absent entry (Beggs [1994]; Bonnano and Vickers [1988]). Firms thus face tradeoffs between the costs of merger (due to lower rival prices) and benefits (due to entry deterrence) that are absent in the monopoly bundling literature. We study these tradeoffs in this paper.

The balance of the paper is organized as follows. Section II presents the general argument for entry-deterring non-horizontal mergers, deriving sufficient conditions for an equilibrium to maximally deter entry. Section III evaluates these conditions in two applications, one a generalized version of Beggs' [1994] model of mergers and malls, and the other a simple model of vertical chains. In both cases, we show that entry-deterring mergers occur despite traditional (contractual precommitment) motives for organizational separation. Section IV concludes. The Appendix contains proofs of formal results stated in the text.
II. THE GENERAL MODEL

Let $N$ denote the number of independent retail units, where a retail unit may be a vertical supplier / producer chain (as in Bonnano and Vickers [1988]) or a multi-product seller (as in Beggs [1994]). Each retail unit may either be merged or not merged. For example, a supplier / producer chain can be vertically integrated (merged) or vertically separated (not merged). Similarly, a multi-product seller can retail each product with independent firms (the case of an unmerged mall) or retail all products in one firm (the case of a merged superstore). The number of merged retail units (or groups) will be denoted by $M$.

For conceptual clarity -- and consonant with the vertical foreclosure literature -- we do not consider horizontal mergers in this paper. Hence, each retail unit is distinct and cannot preempt entry with added "units" (superstores or vertical chains) of its own. For example, when the equilibrium number of retail units is sufficiently small, anti-trust laws preclude horizontal mergers (Hay and Werden [1993]) due to their well-known economic costs (e.g., Farrell and Shapiro [1990]; Werden and Froeb [1998]; Spector [2003]; Cabral [2003]). Section IV discusses implications of horizontal mergers for the paper's conclusions.

Given $N$ and $M$, let $\pi_u(N,M)$ and $\pi_m(N,M)$ denote the equilibrium profit that is earned by an unmerged and merged group, respectively. (We will turn to specific characterizations of these profit functions shortly, but begin by studying their implications for the merger-cum-entry equilibrium.) The following assumption will be maintained (and verified for specific model frameworks) in what follows:

**Assumption 1**: $\pi_u(N,M)$ and $\pi_m(N,M)$ are decreasing in $N$ and $M$ for $0 \leq M \leq N$ and $N \geq 2$.

Due to logic that is well known (e.g., Bonnano and Vickers [1988]; Beggs [1994]), merged groups are more price-competitive, leading to lower equilibrium profits for rival retailers. Because entry increases competition, per-firm profits fall with $N$. 

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Let $E$ denote the cost for a group to enter the market. Entry occurs sequentially with group 1 entering and merging (or not), group 2 entering and merging (or not) next, and so on. Prices, production and trade -- and the realization of profits -- occur after all entry decisions have been made. For simplicity, we adopt the following tie-breaking conventions: (i) if a group is indifferent between entering or not, it does not enter; and (ii) if a group is indifferent between merging or not, it merges. In this sequential game, an entry / merger equilibrium is constrained to be subgame perfect. In addition, for notational simplicity, we define:

\[ \pi(N,M) = \max(\pi^u(N,M), \pi^m(N,M+1)). \]

$\pi(N,M)$ gives the profits of a marginal entrant when, without entry, there are $(N-1)$ operating groups of which $M$ are merged.

Because mergers reduce profits (Assumption 1), entry is "more deterred" when more mergers occur. Hence, if entry is profitable despite $(N-1)$ mergers, with

\[ E < \bar{E}(N) \equiv \pi(N,N-1), \]

then at least $N$ groups will enter. Likewise, if marginal entry is not profitable when $N$ mergers occur,

\[ E \geq \bar{E}(N+1) \equiv \pi(N+1,N) \]

then $N$ mergers will deter entry by the marginal $(N+1)$ group. At this juncture, the following observation is useful, permitting us to focus on marginal entry deterrence without loss in generality:

**Lemma 1.** If marginal entry / expansion (to $(N+1)$ groups) is deterred at $N \geq 2$, then all further entry is deterred.

By Assumption 1, the right-hand-sides of equations (2) and (3) fall with the number of groups $(N)$, $d\bar{E}(N)/dN \leq 0$. Hence, if an equilibrium involves maximal entry deterrence -- minimizing the number of groups subject to entry constraints -- then the equilibrium number of groups is determined by the size of the entry cost $E$. For example, if $E \in [\bar{E}(3), \bar{E}(2)]$, 

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then entry of the third group can be deterred by mergers, and maximal entry deterrence will lead to the operation of just two groups. Similarly, if $E$ is lower, $E \in [\bar{E}(4), \bar{E}(3)]$, then entry of the fourth group (but not the third) can be deterred, and maximal entry deterrence will lead to the operation of three groups. More generally, we can define the unique number of groups that operates with maximal entry deterrence:

$$
N^* \equiv N: E \in [\bar{E}(N+1), \bar{E}(N)].
$$

In order to avoid dwelling on the monopoly case -- wherein merger is trivially optimal for the one group that operates -- we assume:

**Assumption 2**: $N^* \geq 2$.

Within the interval, $[\bar{E}(N+1), \bar{E}(N)]$, different entry costs may require different numbers of mergers in order to deter marginal entry. For example, we can have

$$
\bar{E}(N+1) \leq E < \pi(N+1,N-1) < \bar{E}(N),
$$
in which case $N$ mergers are needed to deter further entry (to $(N+1)$ groups). If $E$ is higher (but still less than $\bar{E}(N)$), then fewer than $N$ mergers will deter entry. We define

$$
M^* \equiv \text{minimum number of mergers needed to achieve maximal entry deterrence (} N^* \text{).}
$$

Knowing $N^*$ and $M^*$, consider the choice problem of an entering group that is *pivotal* to entry deterrence in the following sense:

**Definition**: Suppose that, if it merges, a group anticipates mergers by all subsequent entrants. The group is then *pivotal* if, given the merger decisions of prior entrants, marginal entry (to $(N^*+1)$ groups) is deterred if and only if the present group merges.

If a pivotal group merges, it anticipates subsequent mergers and hence the payoff $\pi^m(N^*,M^*)$, with further entry deterred. However, if it does not merge, then further entry will occur, there is no necessary expectation of subsequent mergers, and the group will
obtain at most $\pi^u(N^*+1,0)$. (By Assumption 1, the unmerged group will obtain less if either more entry occurs or some mergers take place.) Hence, a sufficient condition for maximal entry deterrence to take place is:

**Condition 1. (Strong Entry Effects.)** Due to large costs of entry (vs. merger) in lost profit, groups are better off if they merge and thereby deter marginal entry, than if they do not merge and thereby accommodate marginal entry: $\pi^m(N^*,M^*) \geq \pi^u(N^*+1,0)$.

If Condition 1 holds, then any pivotal group will merge, knowing that subsequent entrants are also pivotal and thus will prefer the entry-deterring merge strategy. Thus, we have:

**Proposition 1.** If Condition 1 holds, then an equilibrium yields maximal entry deterrence, with exactly $N^* \geq 2$ groups operating. 8

However, for some of the cases of interest in this paper -- those for which mergers would not occur absent entry considerations -- groups may prefer not to merge and instead accommodate a marginal unmerged entrant. That is, Condition 1 will be violated. The reason is simple: While unmerged entry depletes profits, the cost of entry-deterring merger - - due to heightened price competition -- can be even greater.

Despite this motive to accommodate unmerged entry, we nevertheless find that groups will often merge to deter entry. The reason is that the groups cannot accommodate an unmerged entrant without either (1) prompting merger by a subsequent entrant (in order to deter yet further entry) or (2) prompting further entry still (opening the floodgates as it were). Merger of a rival negates the only possible advantage of accommodating entry, namely, avoiding the price competition -- and attendant profit cost -- that mergers impart.

To develop these points, consider the following counterpart to Condition 1:
Condition 2A. (Strong Merger Effects.) There is an integer $\beta \geq 1$ such that, due to relatively large costs of merger (vs. entry) in lost profit, groups are better off with $\beta$ fewer mergers, even when this means one more entrant:

\begin{align*}
(6a) \quad & \frac{d\pi_u(N+j,M-\beta j)}{dj} \geq 0, \quad \text{and} \\
(6b) \quad & \frac{d\pi_m(N+j,M-\beta j)}{dj} \geq 0
\end{align*}

for $j \in [0,M/\beta), M \geq \beta$.

For example, when $\beta$ equals one, Condition 2A implies that firms prefer to accommodate a marginal entrant if this means one less merger. In this sense, the equilibrium profit costs of marginal mergers exceed those of marginal entry.

Condition 2A has important implications for the number of mergers required to deter entry. First, evaluating equation (6) at $N=N^*, j \in [0,1), M=N^*-1$ (in equation (6a)), and $M=N^*$ (in equation (6b)), we see that:

\[ \pi(N^*+1,N^*-1-\beta) \geq \pi(N^*,N^*-1) > E, \]

where the last inequality follows from the definition of $N^*$ in equation (4). Hence, if there are only $(N^*-1-\beta)$ merged groups, entry to $(N^*+1)$ groups is profitable; that is, at least $N^*-\beta$ mergers are needed to support the $N^*$ maximal deterrence outcome. Second, with marginal entry, the number of mergers can fall by no more than $\beta$ in order to deter yet further entry. \(^9\)

Now, assuming that Condition 2A holds, consider the choice problem of a pivotal group. If the group does not merge, then further entry will occur (because the group is pivotal). Moreover, for every additional entrant, a “renegade” (non-merging) group can anticipate, at best, only $\beta$ fewer mergers; otherwise, yet more entry occurs. Given this constraint on possible entry-cum-merger outcomes, Condition 2A implies that a renegade prefers more entry and fewer mergers. In other words, the best that a renegade can anticipate is an outcome with no mergers at all and “mega entry” to $N=N^*+M^*/\beta$, yielding him a payoff equal to $\pi_u(N^*+M^*/\beta,0)$. \(^10\) On the other hand, if the group merges, then it deters further
entry and obtains $\pi^m(N^*,M^*)$. The following condition is sufficient for the group to prefer the entry deterrence strategy, despite large costs of merger (Condition 2A):

**Condition 2B. (Costs of Mega Entry.)** Groups prefer to merge, and thereby deter entry, than not merge and thereby accommodate "mega-entry":

$$\pi^m(N^*,M^*) \geq \pi^u(N^*+M^*/\beta,0),$$

where $\beta$ is as defined in Condition 2A.

For example, consider Conditions 2A-2B when $\beta$ equals one. Then (because $M^* \geq N^*-\beta$), maximal deterrence requires that either $M^* = N^*-1$ or $M^* = N^*$ groups merge. In the latter (all merge) case, Condition 2B states that merger, and maximal entry deterrence ($N = N^*$), is preferred to *doubling* the number of firms (to $2N^*$) without merger.

If Conditions 2A-2B hold, then any pivotal group will merge, knowing that subsequent entrants are also pivotal and thus will prefer the entry-deterring merge strategy. Hence, maximal entry deterrence is achieved.

**Proposition 2.** If Conditions 2A and 2B hold, an equilibrium yields maximal entry deterrence, with exactly $N^* \geq 2$ groups operating and at least $(N^* - \beta)$ mergers.

Table I presents a specific numerical example that helps to illustrate the logic underpinning Proposition 2. In this example, up to four groups may enter the market. Table I reveals that profits fall with both entry ($N$) and mergers ($M$), so that Assumption 1 holds. In addition, it is easily verified that

$$\pi^u(N,M) > \pi^m(N,M+1) \Rightarrow \pi(N,M) = \pi^u(N,M),$$

so that no mergers would occur absent entry-deterrence considerations.
Now let us suppose that entry costs are $E = .4$. Then maximal deterrence yields $N^* = 2$ firms, with

$$E = .4 \in [\bar{E}(3), \bar{E}(2)] = [\pi^u(3,2), \pi^u(2,1)] = [50/147, 9/16].$$

If only one firm merges ($M=1$), then entry/expansion to $N=3$ is profitable:

$$\pi^u(3,1) = 25/54 > .4 = E.$$ 

Hence, $M^* = 2$ mergers are needed to limit entry to $N^* = 2$.

The question is: Is (and why is) $(N,M) = (N^*,M^*) = (2,2)$ an equilibrium? We can see that

$$\pi^u(3,0) (= 2/3) > \pi^m(2,2) (= 1/2),$$

so that firms would prefer to accommodate marginal entry/expansion to $N=3$, with no mergers, than to merge and deter further entry. That is, Condition 1 is violated and one might think that firms would therefore prefer not to merge. However, $(N,M) = (3,0)$ is not an option. Why? Because further entry (to $N=4$) is not deterred; that is, $\pi^u(4,0) (= 1/2) > .4 = E$. To deter entry to $N=4$, there must be at least one merger. Hence, $(N,M) = (3,1)$ is an option. In other words, for this example, Condition 2A holds with $\beta = 1$; therefore, at least $M^* - 1 (= 1)$ merger is required to support an $(N^* + 1) (= 3)$ group equilibrium.\(^{11}\) Comparing a “merged” payoff, $\pi^m(N^*,M^*) (= 1/2)$ to either “unmerged” alternative, $\pi^u(3,1)$ or $\pi^u(4,0)$, firms weakly prefer to merge and thereby deter entry; that is, Condition 2B holds, with $\pi^m(N^*,M^*) = \pi^m(2,2) = \pi^m(4,0) = \pi^u(N^* + M^*/\beta, 0) (> \pi^u(3,1))$. Hence, $(N^*,M^*) = (2,2)$ is indeed the equilibrium. Figure 1 completely depicts the extensive form and subgame perfect equilibrium for this entry game, confirming the foregoing logic.

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III. APPLICATIONS

We consider two models of an $N$-firm differentiated product market, one with multi-product retailing and the other with vertical (upstream and downstream) retail chains.
III(i). Mergers and Malls

Beggs [1994] shows that multi-product retailers may prefer to organize themselves as malls - with each product sold by an independent firm -- rather than as merged firms (or superstores) that sell all of the multiple products. We now generalize Beggs [1994] in the simplest possible way to illustrate effects of entry on multi-product mergers. In doing so, we adapt the foregoing results to show that, even when no mergers would occur absent entry-deterrence considerations, the prospect of entry can prompt groups to merge.

Let us suppose that N groups sell two products each. Each group is either merged, selling both products as one firm, or separated, selling the two products as two independent firms. Both goods are produced at constant and zero marginal cost.

Consumer demands are determined in a Hotelling-type environment that has two properties. First, as is standard in the product variety literature (e.g., Kuhn and Vives [1999]; Dixit and Stiglitz [1977]; Spence [1976]), potential products/firms are different from one another in an exogenous and symmetric way. Second, each firm engages in head-to-head competition for customers against all other firms in the market; that is, for any two firms, there are consumers who find these firms' products to be the "best" on offer and, hence, who choose between them. This second property is violated in standard spatial differentiation models wherein consumers and firms have fixed locations on the same line (e.g., Eaton and Wooders [1985]) or circle (e.g., Klemperer [1992]) and, hence, firms compete for consumers only with proximate neighbors.

Specifically, there is a consumer population of size \( \lambda \) and each consumer demands one unit of each of the two products (inelastically). The consumers are uniformly distributed on a circle of circumference \( C \), and the composite products of the \( N \) firms are located at \( N \) equally spaced points on the circle. However, any possible ordering of the \( N \) firms between these locations occurs with equal relative frequency. For example, if there are three firms, then there are six possible orderings of these firms between the three locations on the circle,
and each ordering occurs with a relative frequency of one-sixth. A consumer's "transport" or "preference" cost of purchasing from a particular product location is proportional to her distance from that location (traveling on the circle). This “random preference Hotelling” specification gives rise to the following demand for product $i$ of group $j$:

$$D^i_j = a - b \left[ \left( p^i_1 + p^i_2 \right) - \bar{P}^j \right],$$

where $p^i_1$ and $p^i_2$ are group $j$'s prices for its two products, $\bar{P}^j = \left( N-1 \right) \sum_{h \neq j} (p^h_1 + p^h_2)$ is the average price of group $j$'s competitors, $a=\left( \lambda / N \right) > 0$, $b=\left( \lambda / Ct \right) > 0$, and $t=\text{unit transport cost}$.\(^\text{15}\)

Overall, we call this structure Model 1. For this model, we first determine properties of equilibrium prices and group profits for a given number of operating groups $N$, and a given number of merged groups, $M \leq N$.

Let $p^u$ and $P^m$ denote, respectively, equilibrium prices of an unmerged firm (for its one product) and a merged firm (for its two products). Then we can define the average competitor prices for merged (m) and unmerged (u) groups respectively:

$$\bar{P}^m = (\alpha - k) P^m + (1 - \alpha + k) 2 p^u,$$

$$\bar{P}^u = \alpha P^m + (1 - \alpha) 2p^u,$$

where $k=k(N)=(N-1)^{-1}$ and $\alpha=Mk=M/(N-1)$. Unmerged firms maximize profits as follows (where $i, h \in \{1,2\}$, $h \neq i$):

$$\max_{p^i} \{ a - b \left[ \left( p^i_1 + p^i_h \right) - \bar{P}^u \right] \} \Rightarrow p^i_j = \frac{a - bp^i_h + b \bar{P}^u}{2b}.$$

Similarly, merged firms maximize profits,

$$\max_{p^j} \{ a - b \left[ p^j - \bar{P}^m \right] \} \Rightarrow p^j = \frac{a + b \bar{P}^m}{2b}.$$

Solving (9)-(12) yields the equilibrium prices,

$$p^u = \frac{a(2+k)}{bQ}, \quad P^m = \frac{a(3+2k)}{bQ},$$

where $Q = (2 + \alpha + k)$. We define the attendant equilibrium group profits,

$$\pi^u(N,M) = \left\{ a - b 2 p^u + b \bar{P}^u \right\} 2 p^u = \frac{a^2(2+k)^2}{bQ^2},$$

$$\pi^m(N,M) = \left\{ a - b P^m + b \bar{P}^m \right\} P^m = \frac{a^2(3+2k)^2}{bQ^2}.$$

Several properties of the equilibrium merit note:
Lemma 2. In Model 1, with M mergers and N operating groups, (a) merged prices are lower, $P_m < 2p_u$; (b) merged profits are higher, $\pi^m > \pi^u$; (c) prices and profits fall when the number of merged groups rises, $\partial p_u / \partial M < 0$, $\partial P_m / \partial M < 0$, $\partial \pi^u / \partial M < 0$, and $\partial \pi^m / \partial M < 0$; (d) for $N \geq 2$, an increase in the number of operating firms leads to lower profits, $\partial \pi^u(N,M) / \partial N < 0$ and $\partial \pi^m(N,M) / \partial N < 0$; and hence, (e) Assumption 1 holds.

A merged group considers the adverse effect of an increase in one product's price on the other product's demand. Because an unmerged firm does not consider this effect, it sets a higher price (Lemma 2(a)) at the cost of lower group profit (Lemma 2(b)). Although a merger is directly beneficial to a group, it also heightens price competition by rivals; faced with lower merged group prices, rivals lower their prices, leading to lower market prices and profits (Lemma 2(c)). If the latter (adverse) effect of merger dominates the former (beneficial) effect of merger, groups do not merge in equilibrium (ignoring the entry concerns at issue in the present paper). This is Beggs' [1994] key point, which he develops in a variant of the present model when $N=2$. Indeed, for our model, when $N=2$ and there is no threat of entry, Beggs [1994] shows that neither retail group merges. Generalizing this result to allow for more than two firms, we have: 16

Proposition 3. Given N operating retail groups in Model 1 -- without prospect for further entry -- no firm will merge if $N \leq 5$ and all firms will merge if $N \geq 6$.

The strategic benefit that any given firm derives from organizational separation (a mall structure) wanes as the number of retail groups rises; the reason is that each firm's choice of a mall structure -- and the attendant pre-commitment to higher prices -- has a smaller impact on average rival prices when there are more firms in the market. As a result, when N gets sufficiently large (six in our model), the direct benefits of a merger exceed the strategic
benefits of a mall. However, for sufficiently few firms, the strategic benefits of separation dominate, ignoring entry considerations. These are the cases of interest here.

Even in these cases, we find that mergers will occur when the threat of entry is taken into account. Specifically, an absence of merger -- precisely because it raises prices and profits -- encourages entry of new competitors, which depletes profits (Lemma 2(d)). Conversely, mergers deter costly entry. Despite entry deterrence benefits of mergers, marginal entry is always less costly than entry-deterring mergers in our Model 1. That is, Condition 1 fails to hold. Nevertheless, we have the following:

*Proposition 4.* In Model 1, Conditions 2A and 2B hold (with β=2). Hence, an equilibrium yields maximal entry deterrence, with exactly N*≥ 2 groups operating and at least (N*-2) mergers.

When N*≥ 3, Proposition 4 implies that some mergers occur in order to deter entry. When N*=2, maximal entry deterrence also generally requires one or two mergers. Formally, let us define the parameter

\[ v = (\lambda C_t/E)^{1/2}. \]

*Lemma 3.* In Model 1, N*=int(v-1), where int(X) = minimum integer no lower than X. Hence, N*=2 if and only if \( v \in (2,3] \). For \( v \in (3/2^{1/2}, 2^{1/2}9/5] \approx (2.1213,2.5456] \), M*=1 (and N*=2) and for \( v \in (2^{1/2}9/5,3] \), M*=2 (and N*=2).

Thus, for most values of v that support a two-group equilibrium, there is at least one merger (M*≥ 1) even though, absent entry deterrence considerations, no mergers would occur.

III(ii). *Vertical Integration vs. Vertical Separation*
Bonnano and Vickers [1988] identify strategic advantages of vertical separation in
differentiated product markets. By separating, a vertical chain can sign a contract that
stipulates an above-cost wholesale price, thus implicitly pre-committing the downstream
retailer to charge higher prices (ceteris paribus). Because rivals respond to higher competitor
retail prices by themselves charging higher prices, the contractual pre-commitment is
advantageous to the vertically separated chain. We now reconsider this logic in the simplest
possible model of entry with vertical merger decisions. In doing so, we adapt the results of
Section II to show that the prospect of entry will prompt firms to vertically integrate when
they otherwise would not.

Consider again \(N\) retailers, now with each selling only one product. Products are
supplied by upstream firms. The demand for retailer \(j\)'s product takes the Hotelling form:

\[
D^j = \{ a + b (\bar{P}^j - P^j) \}
\]

where \(P^j\) is retailer \(j\)'s product price, \(\bar{P}^j = \frac{1}{N-1} \sum_{h \neq j} P^h\) is the average price of group \(j\)'s
rivals, \(a = \lambda/N\), and \(b = \lambda/C_t\). Unit costs of production and marketing are constant and zero. A
vertically integrated (merged) retailer thus faces a zero wholesale price. For vertically
separated (unmerged) retailers, we follow Bonnano and Vickers [1988] (and others) in
restricting attention to observable two-part contracts between upstream suppliers and
downstream retailers. Contracts thus stipulate a fixed transfer and a wholesale price that can
be positive, zero or negative (positive in equilibrium). As described in Section II, retailers
(or vertical chains) enter and merge (or not) sequentially. After entry, unmerged chains sign
vertical contracts. Finally, retailers set prices, and production and trade occurs.

We call this structure Model 2, and begin by characterizing equilibrium prices and profits
that prevail with a given number of retail chains (\(N\)) and merged groups (\(M \leq N\):

**Lemma 4.** Given \(M\) and \(N\) in Model 2, unmerged chains sign vertical contracts with the
wholesale price,
(16) \[ w^* = \{ak(2+k)\}/\{b(2+k(1+\alpha))\} > 0, \]
yielding the equilibrium prices and profits,

(17) \[ P_m = (a/b)\{2+2k+k^2\}/\{2+k+\alpha k\} , \quad P_u = (a/b)\{2+3k+k^2\}/\{2+k+\alpha k\}, \]

(18a) \[ \pi_m(N,M) = (a^2/b)\{2+2k+k^2\}^2/\{2+k+\alpha k\}^2, \]

(18b) \[ \pi_u(N,M) = (a^2/b)\{2+3k+k^2\}(2+k)/\{2+k+\alpha k\}^2. \]

**Lemma 5.** In Model 2, with M mergers and N operating groups, (a) merged prices are lower, \( P_m < P_u \); (b) merged profits are higher, \( \pi_m > \pi_u \); (c) prices and profits fall when the number of merged groups (M) rises; (d) for \( N \geq 2 \), entry lowers profits, \( \partial \pi_u(N,M)/\partial N < 0 \) and \( \partial \pi_m(N,M)/\partial N < 0 \); and hence, (e) Assumption 1 holds.

Vertically separated groups precommit to positive wholesale prices that raise their own retail prices (Lemma 5(a)). The benefit of this pre-commitment is that it prompts rivals to raise their retail prices as well. Because vertically integrated firms charge lower prices (ceteris paribus) -- with rivals competing by lowering their prices as well -- prices and profits fall as more groups integrate (Lemma 5(c)). Indeed, this cost of mergers makes them strictly disadvantageous:\(^{18}\)

**Proposition 5.** Given \( N \) operating retailers in Model 2 -- without prospect for further entry -- no firm will vertically integrate in equilibrium. Formally, \( \pi_u(N,M) > \pi_m(N,M+1) \) for \( 0 \leq M \leq N-1, \ N \geq 2 \).

Here, however, vertical integration can be advantageous, precisely because it lowers prices and profits, thereby deterring costly entry (Lemma 5(d)).

**Proposition 6.** In Model 2, Condition 1 holds and, hence, an equilibrium yields maximal entry deterrence, with exactly \( N^* \geq 2 \) retailers operating.
Achieving maximal entry deterrence in Model 2 may or may not require vertical mergers. Because entry has a large adverse effect on profits, it is possible that marginal entry, from $N^*$, is so costly that it is deterred even when there are no mergers. Conversely, however, an equilibrium may require vertical mergers that would not occur absent the entry deterrence considerations of interest here. Formally, recalling the parameter $\nu$ of (equation (15)), let us define the critical values,

$$
\bar{\nu}(N) = \{\nu: M(N; \nu) = N\} = \frac{(2N+2)[N(N+1)]^{1/2}}{2N+1},
$$

$$
\underline{\nu}(N) = \{\nu: M(N; \nu) = 0\} = [N(N+1)]^{1/2},
$$

where $M(N; \nu)$ is defined as the minimum number of mergers required to deter marginal entry. Hence, $\bar{\nu}(N)$ is the level of $\nu$ such that $N$ mergers are required to support an $N$-group equilibrium (setting $\pi_u(N+1,N)$ of equation (18) equal to the entry cost $E$ so that marginal entry is exactly deterred when $M=N$). Similarly, $\underline{\nu}(N)$ is the level of $\nu$ such that no mergers are required to support an $N$-group equilibrium ($\pi_u(N+1,0)=E$). The following Lemma now characterizes the Model 2 equilibrium as a function of $\nu$:

**Lemma 6.** In Model 2, for integers $N \geq 2$: (a) $N < \bar{\nu}(N) < N+1$; (b) if $\nu \in (\bar{\nu}(N-1), \bar{\nu}(N))$, then $N^*=N$; (c) $\bar{\nu}(N-1) < \underline{\nu}(N) < \bar{\nu}(N)$; (d) if $\nu \in (\underline{\nu}(N), \bar{\nu}(N))$, then $M^* \geq 1$ (and $N^*=N$); conversely, if $\nu \in (\bar{\nu}(N-1), \underline{\nu}(N))$, then $M^*=0$ (and $N^*=N$).19

Lemma 6(a)-(b) characterizes the equilibrium number of operating groups ($N^*$) as a function of $\nu$ (given Assumption 2, $N \geq 2$). For values of $\nu$ that support a given $N^*$, Lemma 6(c)-(d) describes when mergers are required to support the maximal deterrence equilibrium ($M^* \geq 1$) and when none are required ($M^*=0$). For example, suppose that $\nu \in (\bar{\nu}(2), \bar{\nu}(3)) \approx (2.939,3.959]$, so that $N^*=3$ groups operate in the equilibrium. Then, if $\nu > \underline{\nu}(3) \approx 3.464$, at least one group merges; and if $\nu > 3.794$, all three groups merge (with $M^*=3$). In summary:
Proposition 7. In Model 2, there are values of \( v \) such that some or all groups vertically integrate in equilibrium.

III(iii). A Note on Welfare in a Random Preference Hotelling Model.

A final issue that we raise here is whether mergers, by deterring entry, raise or deplete economic welfare in Models 1 and 2. To address this issue most simply, we consider symmetric pricing outcomes that send consumers to their closest firm, as is optimal and as occurs in equilibria with all or no mergers. In such cases, because consumer demands are perfectly inelastic, the only economic margin that is relevant for economic welfare is the number of firms (N). On one hand, increasing the number of firms reduces consumer "transport" costs. On the other, it raises set-up / entry costs. Formally, let \( L = (C/N) \) denote the length of the arc between any two product locations (where \( C \) is the circumference of the market circle); and let \( \delta \) denote distance to a product location. Then total consumer transport costs equal:

\[
T = N \int_0^{L/2} t\delta \, d\delta = \frac{tC\lambda}{2N},
\]

where \( t \) is unit transport cost and \( \lambda \) is the total consumer population. An optimal number of firms, \( N^{**} \), minimizes the sum of transport costs \( T \) and entry costs, \( EN \):

\[
N^{**} \in \{\text{int}(n^*), \text{int}(n^*-1)\},
\]

where \( \text{int}(X) = \text{minimum integer no lower than } X \), and

\[
n^* = \arg\min_N \left( \left\lfloor \frac{tC\lambda}{4N} \right\rfloor + EN \right) = \left\lfloor \frac{tC\lambda}{4E} \right\rfloor^{1/2} = \frac{v}{2}.
\]

We wish to compare the optimal number of groups, \( N^{**} \), to the equilibrium number, \( N^* \), and the number of groups that would arise absent any mergers,

\[
N^u = \{\text{minimum integer } N \text{ such that } \pi^u(N+1,0) \leq E\}.
\]

Lemma 7. In Model 1, if \( v > 3/2^{1/2} \approx 2.1213 \) (so that \( N^* \geq 2 \) and \( M^* \geq 1 \), then \( N^u > N^* \geq N^{**} \). In Model 2, for \( v > \bar{v}(1) \) (so that \( N^* \geq 2 \)), \( N^u \geq N^* \geq N^{**} \) and, whenever \( M^* \geq 1 \), \( N^u > N^* \).
In both models, there is excessive entry as firms bid away market rents. Hence, by limiting the extent of excess entry, mergers increase economic welfare. Moreover, the extent to which mergers reduce social costs can be substantial. For example, for Model 1, Table II presents values of N**, N*, and N0 that arise with various different levels of \( \nu \), each of which yields an all-merge equilibrium in unfettered markets; for these cases, mergers have large entry deterrence effects and yield societal cost savings of over 20 percent when compared to equilibria that arise when no mergers are allowed.

PLACE TABLE II ABOUT HERE

Interestingly, in our Model 1, mergers can lead not only to social cost savings, but also to substantial reductions in consumer prices. Even though mergers raise prices by deterring entry, their direct price-dampening impact is larger. In the cases of Table II, for example, mergers lead to net price reductions that range from approximately 17 percent to over 28 percent. Although these effects are not relevant for social welfare here, they suggest that mergers may potentially have a beneficial impact on consumer surplus when consumer demands are not perfectly inelastic.

IV. CONCLUSION

This paper considers how a non-horizontal merger can deter new entry and, for this reason, emerge in equilibrium. Organizational mergers, by implicitly committing firms to more aggressive price competition, can deter entry. By modeling these entry-deterrence effects, this paper shows that non-horizontal mergers can occur in equilibrium even though, absent entry considerations, they do not. We also find that mergers can arise under some rather surprising conditions. One might expect that the likelihood of equilibrium mergers would turn on the relative weight of their pro-competitive (price-dampening) and anti-competitive (entry-deterring) effects. However, we find that mergers occur even when the cost of a
marginal merger (to firms) is greater than the cost of a marginal entrant. The reason is that marginal entry cannot be accommodated, and a marginal merger avoided, without prompting a veritable flood of new rivals.

To the extent that entry or exit is possible in multiple-good retail markets, we thus find a strategic motive for the emergence of the "big box" superstores so prevalent in contemporary retail markets. For example, there are many complaints that Walmart promotes the exit of local competitors (PBS [2001]); this paper suggests that Walmart's merged structure may find motive in precisely this effect. Of course, we ignore some considerations that may also affect the equilibrium choice between malls and superstores. Malls may be advantageous when retail products require customer service, with decentralized ownership permitting better incentives for service provision. On the other hand, there may be cost economies that favor superstores.

This paper likewise identifies a new strategic motive for vertical integration. While the extant literature on vertical integration is extensive, this phenomenon is explained by either cost economies, or benefits of avoiding double-marginalization, or incomplete contracts for relationship-specific investments (Grossman and Hart [1986]), or incentives to prevent downstream horizontal integration (Conagelo [1995]), or attempts at vertical foreclosure -- when a chain can deny its rivals access to upstream production (e.g., Hart and Tirole [1990]). For conceptual clarity, none of these forces operate in the present paper. Instead, we find that vertical integration can occur because it credibly deters the entry of prospective rivals. The logic of this result requires that vertically separated chains can sign mutually advantageous contracts that have the intended effect of limiting competition. If anti-trust laws preclude vertical restraints, then our entry-deterrence motive for vertical integration evaporates. To our knowledge, contemporary anti-trust laws in the developed world do not prevent the two-part vertical contracts considered here. However, as a matter of policy, one might ask: Should such vertical restraints be limited? Alternately, should vertical integration be proscribed?

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In the context of our specific model applications, the answers to such questions are quite clear. Mergers can raise economic welfare by reducing the extent of excess entry. Indeed, in our Model 1, mergers lead to lower consumer prices even though they substantially reduce the number of firms. These outcomes suggest that entry-deterring mergers may potentially enhance consumer surplus in more general models that allow for elastic consumer demands. However, caution is advised in interpreting these conclusions; they arise in the specific frameworks considered here and need not be general.

Perhaps the greatest limitation of our analysis is its neglect of any possible horizontal coordination. However, our arguments are likely to be robust. Consider, for example, an incumbent monopolist who can set up N "firms" (as subsidiaries) before entry takes place (a standard assumption in the horizontal merger literature). It will be advantageous for the incumbent to deter further entry. This can be done in two ways. First, the incumbent can sign horizontal franchise contracts with subsidiaries that implicitly coordinate the franchisees in the absence of entry, but permit open decentralized competition in the event of entry (Hadfield [1991]). Second, in order to achieve maximal entry deterrence, the franchisees can be merged (vertically or across products) in order to implicitly precommit them to more aggressive competition when entry occurs. In sum, because they limit entry, non-horizontal mergers continue to arise.
APPENDIX

Proof of Lemma 1. It suffices (by induction) to show that if marginal entry to \((N+1)\) firms is deterred, \(\pi(N+1,M) \leq E\), then entry to \((N+2)\) firms is deterred:

\[
\pi(N+2,M) \leq \pi(N+1,M) (\leq E), \text{ and } \pi(N+2,M+1) \leq \pi(N+1,M) (\leq E).
\]

Considering the two possible cases, \(\pi^u(N+1,M) \geq \pi^m(N+1,M+1)\) and \(\pi^m(N+1,M+1) \geq \pi^u(N+1,M)\), the latter inequalities can be seen to follow directly from Assumption 1. QED.

Proof of Proposition 2. Define

\[M(N) = \text{minimum number of mergers needed to deter further entry / expansion (to \((N+1)\) groups) } = \{M: \pi(N+1,M) = E\}.
\]

Expanding the derivatives in equation (6) (and recalling equation (1)), we have:

Lemma A1. If Condition 2A holds, then \(dM(N)/dN \geq -\beta\) for \(N^* \leq N \leq \bar{N}\), where \(\bar{N} = \{N: M(N) = 0\}\).

Proof of Lemma A1. First note that

(A1) \[dM(N)/dN = \delta (dM^u(N)/dN) + (1- \delta) (dM^m(N)/dN),\]

where \(M^u(N) = \{M: \pi^u(N+1,M) = E\}\), \(M^m(N) = \{M: \pi^m(N+1,M) = E\}\), and \(\delta \in \{0,1\}\).

Expanding the derivatives in (6) gives

(A2) \[-\beta \leq (-\partial \pi^z(N+1,M)/\partial N)/(-\partial \pi^z(N+1,M)/\partial M) \quad \text{for } z \in \{u,m\}.
\]

Evaluated at \(M=M^z(N)\), the right-hand-side of (A2) equals \(dM^z(N)/dN\) for \(z \in \{u,m\}\); hence, (A1)-(A2) imply that \(dM(N)/dN \geq -\beta\). QED Lemma A1.

Lemma A2. If Condition 2A holds, then (a) \(M^* \geq N^* - \beta\), and (b) for integers \(j \in \{1,\ldots,J\}\), at least \(M^*-\beta j\) mergers are needed to support an \((N^*+j)\) group equilibrium (deterring entry / expansion to \((N^*+j+1)\) groups), where \(J\) is the maximum integer such that \(M^*-\beta J \geq 0\).
Proof of Lemma A2. (a) See text. (b) Define $M^*(N)=\text{int}(M(N))=$minimum number of mergers that can support an $N$-group equilibrium, where $\text{int}(X)=\text{minimum integer not less than } X$. By recursion, Lemma A2(b) follows if $M^*(N+1)\geq M^*(N)-\beta$. By Lemma A1 (and Condition 2A), we have $M(N+1)\geq M(N)-\beta$, which implies

$$M^*(N+1) = \text{int}(M(N+1)) \geq \text{int}(M(N)-\beta) = \text{int}(M(N)) - \beta = M^*(N) - \beta,$$

where we use the definition of $\beta$ as an integer. QED Lemma A2.

Consider a pivotal group. If it merges, then it anticipates the payoff $\pi^m(N^*,M^*)$. If it does not merge, then (by Lemma A2(b)) it can anticipate at best an equilibrium with $(N^*+j)$ operating groups and $(M^*-\beta j)$ mergers for some $j\geq 1$ (and $j\leq M^*/\beta$). Among these potential equilibria, Condition 2A implies that the group prefers the highest $j$ ($j=M^*/\beta$), “mega-entry” (with $N=N^*+j=N^*+M^*/\beta$), and no mergers (with $M=M^*-\beta j=0$). Hence, an upper bound on the group’s “no-merge payoff” is $\pi^u(N^*+M^*/\beta,0)$. (If $M^*/\beta$ is not an integer, then the non-merging group obtains less.) Comparing the best possible “no merge payoff,” $\pi^u(N^*+M^*/\beta,0)$, and its “merge” counterpart, $\pi^m(N^*,M^*)$, Condition 2B implies that the pivotal group prefers to merge. QED.

Proof of Lemma 2. (a)-(b) follow directly from (13)-(14). (c) follows from $\partial Q/\partial \alpha=1>0$, $\alpha=Mk$, and (13)-(14). To establish (d), we can use (14) to obtain

(A3) $\partial \pi^u(N,M)/\partial N \overset{S}{=} \Delta^u(N,M) \equiv -(2+k)Q(N-1)^2 - NQ + (2+k)N\{M+1\},$

where $Q=(2+\alpha+k)$ and "=S" denotes "equals in sign." Now note:

(A4) $\partial \Delta^u(N,M)/\partial M = 1 > 0.$

By (A3)-(A4), it suffices to sign $\Delta^u()$ when $M=N-1$ (so that $Mk=1$).

(A5) $\Delta^u(N,N-1) = (2+k)(2N-1)\{2-N\} - N(3+k) < 0 \text{ for } N\geq 2.$

Turning next to $\pi^m()$,

(A6) $\partial \pi^m(N,M)/\partial N \overset{S}{=} \Delta^m(N,M) \equiv -(3+2k)Q(N-1)^2 - 2NQ + (3+2k)N\{M+1\},$

Now note:

(A7) $\partial \Delta^m(N,M)/\partial M = 1 > 0.$
By (A6)-(A7), it suffices to sign $\Delta^m$ when $M=N$. Substituting for $Q$ and $k$ in (A6) and rewriting, we find: $\Delta^m(N,N) = -(3+2k)(2N-1)(N-1) < 0$ for $N\geq 2$. We thus have $\Delta^u(N,M)<0$ and $\Delta^m(N,M)<0$ for all $M\leq N$ and all $N\geq 2$, implying part (d) (by (A3) and (A6)). QED.

**Proof of Proposition 3.** Define the unilateral benefit of separation (not merge):

$$\pi^u(N,M) - \pi^m(N,M+1) \overset{S}{=} \Delta(k,\alpha) \equiv -4+12k+31k^2+20k^3+4k^4 - [4\alpha+2k\alpha+\alpha^2(1+4k+2k^2)]$$

where $k=(N-1)^{-1}$, $\alpha=Mk$, and $0\leq M\leq N-1$. If $\Delta>0$ for all $\alpha\in[0,1]$, as is easily verified for $N\leq 3$, no firms merge in a subgame perfect equilibrium. Likewise, if $\Delta<0$ for all $\alpha\in[0,1]$, as is easily verified for $N\geq 6$, then all firms merge in a subgame perfect equilibrium. This leaves two cases, $N=4$ and $N=5$. For $N=4$, we have $\Delta>0$ for $\alpha\in\{0,1/3\}$, and $\Delta<0$ for $\alpha\in\{2/3,1\}$. For $N=5$, we have $\Delta>0$ for $\alpha=0$, and $\Delta<0$ for $\alpha\in\{1/4,1/2,3/4,1\}$. In both cases, if we construct the extensive forms for the sequential move (merge) game, and solve by backward induction, these $\Delta$ values -- combined with Lemma 2(e) / Assumption 1 -- imply a unique equilibrium in which no firm merges. For example, for $N=4$, let us order the firms by order of play from F1 to F4. Except when at least two prior firms merge (action m), F4 does not merge (action u) because $\Delta>0$. Now consider F3’s choice when at most one of the prior firms has merged. If F3 chooses u, then F4 will follow suit and F3 thus obtains $\pi^u(4,M)$, with $M\leq 1$ denoting prior firm mergers. If F3 chooses m, then it obtains at most $\pi^m(4,M+1)$ ($>\pi^m(4,M+2)$ by Lemma 2(e))). With $\alpha\leq 1/3$ and, hence, $\Delta>0$, F3 thus chooses U. Hence, if F2 chooses u, F3 and F4 choose u, yielding F2 $\pi^u(4,M)$, $M\leq 1$; if F2 chooses m, then it obtains at most $\pi^m(4,M+1)$. Because $\alpha\leq 1/3$, F2 chooses u. Finally, F1 anticipates u by F2-F4 and thus chooses u (because $\alpha=0$ and, hence, $\Delta>0$). Similar logic applies to $N=5$. QED.

**Proof of Proposition 4.** First, to verify Condition 2A for Model 1, we have (for $\beta=2$, from equation (14)):

(A8a) \[ \pi^u(N+j,M-2j) = \{2\lambda C/(2N+M-1)^2\} \{2-(N+j)^{-1}\}^2 \]

(A8b) \[ \pi^m(N+j,M-2j) = \{\lambda C/(2N+M-1)^2\} \{3-(N+j)^{-1}\}^2 \]
Equation (A8) directly implies the inequalities in equation (6). Second, to verify Condition 2B (for β=2), we have:

\begin{equation}
\pi^m(N,M) - \pi^u(N+(M/2),0) \overset{S}{=} (3N-1)^2(2N+M)^2 - 8N^2(2N+M-1)^2
\end{equation}

\begin{equation}
= (4N^4 + 4N^3M - 8N^2M - 6NM^2) + (8N^3 - 4N^2) + (N^2M^2 + 4NM + M^2) > 0
\end{equation}

for \(N \geq 2\), where the inequality follows from \(M \leq N\), which implies that the first bracketed term in (A9) is positive. Proposition 4 now follows from Proposition 2. QED.

**Proof of Lemma 3.** \(N^* = \text{int}(v-1)\) follows from the definition of \(N^*\),

\[ N^* = \{\text{minimum integer } N \text{ such that } \pi(N+1,N) \leq E\}, \]

and the observation that, for \(N \geq 2\) in Model 1, \(\pi(N+1,N) = \pi^m(N+1,N+1) = (v(N+1))^2 E\).

For \(v \in (2,3]\), \(N^* = 2\) (from above). Assuming \(v \in (2,3]\), \(M^* \geq 1\) when zero mergers yields entry (to \(N=3\)), \(\pi(3,0) = \pi^u(3,0) = 2v^2E/9 > E \leftrightarrow v > 3/2^{1/2}\). Similarly, \(M^* = 2\) when only one merger yields entry (to \(N=3\)), \(\pi(3,1) = \pi^u(3,1) = 5^2v^2E/(9^22) > E \leftrightarrow v > 21/29/5\). QED.

**Proof of Lemma 4.** For reasons that will become clear shortly, we partition the groups into those that are merged (m), an unmerged chain (j), and unmerged groups other than j (u).

Denoting the corresponding retailer prices by \(\{P^m, P^j, P^u\}\), respective average competitor prices are:

\begin{equation}
P^m = (\alpha-k) P^m + (1-\alpha) P^u + k P^i, \quad P^u = \alpha P^m + (1-\alpha-k) P^u + k P^i, \quad P^j = \alpha P^m + (1-\alpha) P^u,
\end{equation}

where, as before, \(k = k(N) = (N-1)^{-1}\) and \(\alpha = Mk = M/(N-1)\). Denoting contractual wholesale prices for group j and other unmerged groups (u) by \(w^j\) and \(w^u\), respectively, unmerged firms maximize profits as follows:

\begin{equation}
\max_{P^h} \{ a + b (P^h - Ph) \} (P^h - w^h) \Rightarrow P^h = \frac{a + b(P^h + w^h)}{2b}.
\end{equation}

for \(h \in \{j,u\}\). Similarly, merged firms maximize profits,

\begin{equation}
\max_{P^m} \{ a + b (P^m - P^m) \} P^m \Rightarrow P^m = \frac{a + bP^m}{2b}.
\end{equation}

Solving (A10)-(A11) for \(\{P^m, P^j, P^u\}\) gives:
\[(A12a)\quad P_j(w_j, w_u) = \{a(2+k)+bw_j(1+k)+bw_u(1-\alpha)\}/\{b(2+k)\}\]
\[(A12b)\quad P_u(w_j, w_u) = \{a(2+k)+bkw_j+bwu(2-\alpha)\}/\{b(2+k)\}\]
\[(A12c)\quad P_m(w_j, w_u) = \{a(2+k)+bkw_j+bwu(1-\alpha)\}/\{b(2+k)\}\]

The vertically separated chain \(j\) selects its contractual wholesale price \(w_j\) to maximize its integrated profit as follows:

\[(A13)\quad \max_{w_j} \left\{ a + b(\bar{P}_j - P_j(w_j, w_u)) \right\} P_j(w_j, w_u),\]

where \(\bar{P}_j = \alpha P_m(w_j, w_u) + (1-\alpha) P_u(w_j, w_u)\). Solving (A13) and setting \(w_j=w_u\) (by symmetry) yields the equilibrium wholesale price, for vertically separated chains, given in equation (16). Substituting into price and profit functions gives equations (17)-(18). QED.

Proof of Lemma 5. (a)-(c) follow directly from equations (17)-(18), with \(\alpha=Mk\). For (d), beginning with \(\pi_u()\) (evaluated at \(\alpha=Mk\)), we have:

\[(A14)\quad \frac{\partial \pi_u()}{\partial N} = (A+B+C+D)(\pi_u/(N-1)^2)\quad A = -2(N-1)^2/N < 0\quad B = -1/(2+k) < 0\quad C = -(3+2k)/(2+3k+k^2) < 0\quad D = 2(1+2\alpha)/(2+k+\alpha k) > 0.\]

Noting that \(\partial D/\partial \alpha > 0\) and \(\alpha \leq 1\) (with \(M \leq N-1\) when there is an unmerged firm), it suffices to show that \(\partial \pi_u()/\partial N < 0\) at \(\alpha=1\). Moreover, at \(\alpha=1\),

\[(A15)\quad A+D = s -2(N-1)^2(2+2k)+6N \leq 0\quad \text{for } N \geq 3.\]

(A15) implies that \(\partial \pi_u()/\partial N\) in (A14) is negative for \(N \geq 3\). For the remaining case (\(N=2\)), we have (evaluating (A14) at \(\alpha=1\))

\[\frac{\partial \pi_u(2,M)}{\partial N} \leq -(2/3) \pi_u() < 0.\]

Turning next to \(\pi_m()\), we have (with \(A\) and \(D\) as in (A14)):

\[(A16)\quad \frac{\partial \pi_m()}{\partial N} = (A+D+E)(\pi_m/(N-1)^2)\quad E = -(4+4k)/(2+2k+k^2) < 0.\]

Noting that \(\partial D/\partial \alpha > 0\) and \(\alpha \leq 1+k\), we have (evaluating (A16) at \(\alpha=1+k\)),

\[(A17)\quad (A+D+E) \leq A + 2/\{2+2k+k^2\} = s -2(N-1)^2 - (N-1) < 0\quad \text{for } N \geq 2.\]

(A17) implies that \(\partial \pi_m()/\partial N\) in (A16) is negative for \(N \geq 2\). QED.
Proof of Proposition 5. Using equation (18) and the definition of $\alpha$, we have
\[
\pi^u(N,M) - \pi^m(N,M+1) = \pi^u(N,\alpha(N-1)) - \pi^m(N,(\alpha+k)(N-1)) \\
\overset{S}{=} (2+3k+k^2)(2+k)(2+k+\alpha k+k^2)^2 - (2+2k+k^2)^2(2+k+\alpha k)^2 \\
= 4k^2+k^3(16-4\alpha) + k^4(21-2\alpha-3\alpha^2) + k^5(15-3\alpha^2) + k^6(6-\alpha^2)+k^7 > 0,
\]
where the inequality is due to $\alpha \leq 1$ (with $M \leq N-1$ when there is an unmerged firm). QED.

Proof of Proposition 6. Using equation (18), we have
\[
\pi^m(N,N) - \pi^u(N+1,0) \overset{S}{=} \{2N^2 - 2N + 1\}/(N^2(N-1)^2) - \{N(N+1)\}^{-1} \\
\overset{S}{=} N^2 (N+1) + (N-1)^2 > 0.
\]
Hence, Condition 1 is satisfied and, by Proposition 1, maximal entry deterrence occurs. QED.

Proof of Lemma 6. (a) and (c) follow from the definitions of $\bar{v}(N)$ and $\gamma(N)$. To establish (b), let us define, for the continuous variable $n$,
\[
N^+ = \{n: M(n;v) = n \leftrightarrow \bar{v}(n)=v\}.
\]
Recalling the definition of $N^*$, $N^*=$int($N^+$) is the minimum $N$ such that $N$ mergers deters further entry (formal proof available from the author); that is, if $N^+ \in (N-1,N]$, then $N^*=N$. With $N^+ \in (N-1,N]$ iff $v \in (\bar{v}(N-1), \bar{v}(N)]$, (b) follows. Finally, (d) follows from (i) the definition of $M^*=$int($M(N^*;v)$), and (ii) the definition of $\gamma(N)$, which implies that int($M(N;v)$)$=0$ when $v<\gamma(N)$ and int($M(N;v)$)$\geq 1$ when $v>\gamma(N)$. QED.

Proof of Lemma 7. From the definitions of $N^u$, $N^*$ and $M^*$ (and satisfaction of Assumption 1), $N^u > N^*$ when $M^* \geq 1$ and $N^u = N^*$ when $M^* = 0$. By Proposition 4 and Lemma 3, $M^* \geq 1$ when $v>3/2^{1/2}$ in Model 1, implying that $N^u > N^*$. Comparing $N^*$ and $N^{**}$ in Model 1, we have, for $v>2$, $N^* = \text{int}(v-1) \geq \text{int}(v/2) \geq N^{**}$. To establish that $N^* \geq N^{**}$ in Model 2, note (from Lemma 4, Proposition 5, and the definition of $v$),
\[
(A18) \quad N^* = \text{int}(n), \text{ where } n \text{ solves: } \pi^u(n+1,n)/E-1 = \{v^2 (2n+1)^2\}/\{n(n+1)(2n+2)^2\} - 1 = 0.
\]

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Note further that if \( n \geq \nu/2 \), then \( N^* = \text{int}(n) \geq \text{int}(\nu/2) \geq N^{**} \), the desired result. Hence, with \( d\pi^u(n+1,n)/dn < 0 \), it suffices to show that \( \pi^u(n+1,n)/E \geq 1 \) at \( n=(\nu/2) \) (for \( \nu > \bar{\nu}(1) \approx 1.8856 \)), which follows immediately from (A18). QED.
REFERENCES


<table>
<thead>
<tr>
<th>Number of Mergers M</th>
<th>( \pi^u(N,M) )</th>
<th>( \pi^n(N,M) )</th>
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<tr>
<td>( N )</td>
<td>( M )</td>
<td>( M )</td>
</tr>
<tr>
<td>2</td>
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<td>2</td>
</tr>
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<td></td>
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<td>9/25</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
</tr>
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<td>50/104</td>
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<tr>
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<td>50/147</td>
<td>50/192</td>
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<tr>
<td></td>
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</tr>
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\( \pi^u(N,M) \) and \( \pi^n(N,M) \) represent different metrics of the number of mergers in a four-group setup.
Figure 1. Extensive Form for Four-Group Game

\[ u: \pi^u(3,1) \quad m*: \pi^m(3,1) \quad u*: \pi^u(3,1) \quad m*: \pi^m(3,1) \quad u*: \pi^u(3,1) \quad m*: \pi^m(3,1) \]

A Strategies are denoted by “m” and “u” for “merge” and “unmerge.” At each node, the decision-maker is indicated by F1 (for firm / entrant 1) to F4 (for firm / entrant 4). On each branch is given the decision-maker’s payoff from that branch / strategy in view of subgame perfect strategies of subsequent entrants. Branches end when subsequent entry is deterred. Subgame perfect strategies are denoted by asterisks and represent the branch /strategy that yields the highest decision-maker payoff.
Table II. Optimal and Equilibrium Entry in Model 1

<table>
<thead>
<tr>
<th>Value of Parameter $\nu$</th>
<th>Optimal Number of Entrants $N^{**}$</th>
<th>Equilibrium Number of Entrants $N^*$</th>
<th>Number of Entrants with No Mergers $N^u$</th>
<th>Percentage Price Reduction Due to Mergers$^A$</th>
<th>Proportionate Societal Cost Reduction Due to Mergers$^B$</th>
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<td>4</td>
<td>2</td>
<td>3</td>
<td>5</td>
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<td>25.3%</td>
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<td>3</td>
<td>5</td>
<td>8</td>
<td>20.0%</td>
<td>25.5%</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>7</td>
<td>11</td>
<td>21.4%</td>
<td>25.4%</td>
</tr>
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<td>5</td>
<td>9</td>
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<td>22.2%</td>
<td>25.4%</td>
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<tr>
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<td>50</td>
<td>99</td>
<td>141</td>
<td>28.8%</td>
<td>21.7%</td>
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</tbody>
</table>

$^A$ When there are no mergers, equilibrium group prices are $2p^u=2Ct/N^u$. Whenever $\nu$ is an integer, all groups merge in the merger equilibrium, $M^*=N^*$, and group prices are $P^m=Ct/N^*$. The percentage price reduction reported here is $(1-(P^m/2p^u))= 1-(N^u/2N^*)$.

$^B$ Proportionate cost reductions reflect the change in cost when moving from $N^u$ to $N^*$, as a fraction of costs with $N=N^u$. 
FOOTNOTES

1Some recent papers (Pashigian [1998]; Bruekner [1993]) consider how mall contracts (and store rents in particular) can help internalize inter-store externalities in the choice of store space (square footage). This work is particularly useful in showing how mall developers can maximize and reap mall profits. Other papers study the economics of multi-product retailing (e.g., Bliss [1988]; Klemperer [1992]; Armstrong [1999]; Girard-Heraud, et al. [2003]; and many others). However, neither of these literatures considers the choice between alternative organizational forms (mall vs. superstore) and the pricing externalities at the heart of this choice.

2There is an extensive literature on optimal contracts in vertically separated markets; a very small sample -- with apologies for omissions -- is Rey and Stiglitz [1988], Bolton and Bonnano [1988], Wiinter [1993], Perry and Besanko [1991], Blair and Lewis [1994]. To our knowledge, the entry-enhancing effect of vertical contracts -- and the attendant entry-deterrence motive for vertical integration identified here -- have not yet been studied. The vertical restraint literature implicitly offers some other motives for vertical integration that are absent here. For example, contracts may be unable to achieve desired outcomes due to their unobservability (O'Brien and Shaffer [1992]) or a principle's limited commitment ability (McAfee and Schwartz [1994]).

3See also related papers on entry deterrence by exogenously vertically integrated firms that can affect rivals' input supplies (Song and Kim [2000]; Reiffen [1998]; Weisman [1995]) and on supply assurance motivations for vertical integration (Bolton and Whinston [1993]). Chemla [2003] presents an extreme version of the raising rivals' cost argument, wherein vertical integration by an upstream monopolist forecloses downstream competition that would otherwise arise due to exogenous limits on the monopolist's contractual bargaining power.

4See also Nalebluff [2000, 2004b]. These papers focus on the effects of commodity bundling in Cournot markets, but not on entry deterrence.

5For example, in our “merger vs. mall” model, there is exogenous commodity bundling in the sense that customers buy both of two products from one outlet; however, different outlets compete in both products and there is no monopoly in either.


7We assume that E is the same, whether an unmerged group enters or a single merged group enters. We thus avoid an obvious motive for merger -- the saving of setup/entry costs.
Our tie-breaking assumption (that an indifferent firm merges) ensures that the equilibrium is unique. For example, assuming that $\pi^M(N,M) \geq \pi^U(N,M)$ (as is implied by revealed preference), then the first $M^*$ entrants merge in an equilibrium (because merged firms obtain weakly higher profit than do unmerged firms). If $M^* < N^*$, then the remaining $(N^*-M^*)$ firms play a sequential merger game that has a unique subgame perfect equilibrium.

See Appendix, Lemma A2.

It is possible that, with no mergers, entry will occur beyond $N^*+M^*/\beta$, in which case the renegade (non-merging) group may obtain less than $\pi^U(N^*+M^*/\beta,0)$. Consonant with this possibility, our argument requires that $\pi^U(N^*+M^*/\beta,0)$ be an upper bound on the renegade’s anticipated payoff.

The example satisfies the integer analog of Condition 2A, $\pi_z(N+1,M-\beta) \geq \pi_z(N,M)$, $z \in \{u,m\}$, for integers $N \geq N^*$, $M \geq \beta$. Without loss, the example profit functions can be made to satisfy the derivative Condition 2A by the construction, $\pi_z(N+j,M-\beta j) = \pi_z(N,M) + [\pi_z(N+1,M-\beta) - \pi_z(N,M)] j$, for $z \in \{u,m\}$, $j \in (0,1)$, and $\beta=1$.

Our premise of exogenous and symmetric market location for different firm products also parallels recent work on spatially differentiated markets (e.g., Girard-Heraud, Hamudi and Mokrane [2003]). Allowing for endogenous product location decisions (as is the focus of Heywood, et al. [2001], for example) would dramatically complicate our analysis, but is unlikely to qualitatively alter our conclusions. Non-horizontal mergers will intensify price competition among operating sellers, but deter entry and thereby expand the market segment that each seller serves.

While this second property is quite realistic, it is also useful for our analysis because it permits us to derive firm decisions that are a function of market-wide merger activity. In recent work, Chen and Riordan [2005] construct a Hotelling-type “spokes” model that also has this property.

For example, suppose that there are four products, A, B, C, and D. Suppose further that consumer 1 most prefers A and B, and consumer 2 most prefers A and C. Then, in the standard model, consumer 3 cannot most prefer A and D, and firm A only directly competes with firms B and C.

The number of customers served by retail group $j$ in this model (and, hence, the demand for product $i$ in group $j$’s “store”) equals $\sum_{k \neq j} d_k^j q_k^i$, where $d_k^j = 2/(N-1)$ is the relative frequency with which firm $j$ has firm $k$ as a
neighbor, \( d_{jk}^j = \frac{[(Ct/N) + P_k - P_j] \lambda}{2Ct} \) is firm \( j \)'s consumer demand on the arc between firms \( j \) and \( k \) when the two are neighbors, \( P_h = p_{1h} + p_{2h} \) denotes the composite firm \( h \) product price, and the arc length is equal to \( C/N \).

Substitution yields equation (8) with the indicated parameter values. Beggs [1994] studies a linear demand with distinct coefficients on own price, coefficient \( b \) (on \( P_j = p_{1j} + p_{2j} \)), and competitors' price, coefficient \( d \) (on \( P_j \)). Assuming that own-price effects dominate (\( b \geq d \)), he shows that the incentive not to merge is greatest when \( b = d \). We focus on the latter (\( b = d \)) case for three reasons: simplicity, because it follows from a Hotelling-type specification, and because we wish to show that mergers arise -- due to entry-deterrence considerations -- when they otherwise would not. We also note, as in Beggs [1994], that the equation (8) demands arise as the limiting case for a class of quadratic preferences (see expanded paper for details).

The outcomes described in Proposition 3 are unique subgame perfect equilibria in the sequential move (merge) game modeled in this paper. They are also Nash equilibria in a simultaneous move game. When \( N \leq 3 \) or \( N \geq 6 \), the simultaneous move equilibria are unique. However, when \( N = 4 \) or \( N = 5 \), there are two simultaneous move equilibria, all-merge and all-not-merge, with the latter yielding higher firm profits.

See, for example, Ziss [1995], Shaffer [1991], Mathewson and Winter [1984], Hamilton [2003] for other analyses of two-part contracts. For analyses of unobservable contracts, see O'Brien and Shaffer [1992] and McAfee and Schwartz [1994]. For a critique of two-part contracts in vertically separated markets, see Fershtman, Judd and Kelai [1991].

The logic of Bonanno and Vickers [1988] implies that this property is general. A vertically separated chain can achieve the same outcome as an integrated firm by contracting for a zero wholesale price; however, the vertically separated retailer prefers to select a positive wholesale price, implying (by revealed preference) that integration is strictly disadvantageous.

It is easily seen (given Proposition 5 and Lemma 5) that the unique subgame perfect equilibrium to the sequential entry game of Model 2 involves vertical mergers by the first \( M^* \) entrants, and vertical separation by the subsequent \( N^*-M^* \) entrants.

By Lemma 3, Assumption 2 requires \( v > 2 \) in order for \( N^* \geq 2 \) in Model 1. Similarly, by Lemma 6(b), Assumption 2 requires \( v > v(1) \) in order for \( N^* \geq 2 \) in Model 2.
In all-merge equilibria, Lemma 7 implies that mergers (vs. no mergers) increase welfare. In equilibria with some (but not all) firms merging, the welfare gains from mergers (due to entry deterrence) are partially offset by transport cost inefficiencies due to asymmetric product pricing.

In Model 2, all-merge equilibria also lower prices (vis-à-vis no-merge outcomes). Specifically, for \( N^* \geq 3 \), all-merge equilibrium prices are 18 to 33 percent lower than no-merge prices that prevail with one additional entrant \( (N = N^* + 1) \); when \( N^* = 2 \), all-merge \( (N = N^*) \) and no-merge \( (N = N^* + 1) \) prices are identical. However, partial-merge equilibria (when only a subset of firms merge to deter entry) can yield higher average prices than no-merge counterparts.