This paper studies an enforcement game between a regulator and firms that can cause harmful accidents. The distribution of potential accident damage is private information to the firms, and realized damage can be observed only at the cost of going to court. Under conditions described in the paper, an optimal policy involves the separate assessment of regulatory/settlement fines and court liability. In this optimum, injurers self-select by appealing (or not) to the court process; liability takes a ‘threshold’ form, assessing maximal liability when damages are high and zero liability otherwise; and, vis-à-vis a first-best, some firms are over-deterred—and others under-deterred—from having accidents.

INTRODUCTION

When public resources are damaged in an industrial accident, chemical release, oil spill or other pollution event, the government can impose fines on the injuring firm; in addition, both the government and the injuring firm can turn to the courts for assessment of liability. Enforcement mechanisms are important because they provide incentives for firms, ex ante, to undertake precautionary/care measures that reduce the likelihood of an accident. However, given their costs, why use the courts to impose sanctions, rather than simple regulatory fines? When and for whom should the court process be employed? And, given enforcement costs, how should penalties be structured?

Motivated by observed realities in regulatory law enforcement, we study these questions in a model with (1) stochastic damages from a firm’s accident, (2) heterogeneous firms, some of whom have more harmful accidents than others, and (3) a costly court process that can determine the realized level of damage from an accident and assign liability. In this setting, we find when and how regulatory fines and appeals to the courts can be optimally used to sort injurers. Economic costs of this sorting mechanism are shown to be minimized by court-imposed penalties that take a ‘threshold’ form, maximizing a firm’s payment obligations when realized damages are above a given threshold, and otherwise assigning zero liability.

While related to extensive literatures on tort liability, the legal process and law enforcement, this paper’s analysis of public enforcement is distinct in a number of respects. Recent papers consider the optimal design of liability in view of general stochastic damages (Lewis and Sappington 1999, 2001; Innes 1999a), finding that a threshold rule can optimally mitigate the problem of a ‘disappearing defendant’ whose assets are often exceeded by accident damage. Here, in contrast, this payment structure is motivated by costs of law enforcement. Indeed, in this paper injurer assets may exceed maximum possible harm from a firm’s activities; hence injurers can be confronted with strict
liability for harm caused, clearly vitiating any limited liability motive for threshold penalties. Costs of law enforcement, on the other hand, might be thought to motivate a Becker (1968) rule under which liability is uniformly maximal in order to minimize the probability of costly prosecution required to achieve a requisite accident deterrent. However, whenever enforcement costs are borne in this paper, injuring firms are completely exempted from penalty if they have caused sufficiently little damage. The reason is that some absence of liability is necessary for the legal rule to distinguish between injurers with different damage distributions, a differentiation that is the sole economic purpose of the costly enforcement/court process modelled here. Thus, we find a new exception to Becker’s (1968) prescription of maximal sanctions. Moreover, this exception withstands various generalizations of the paper’s model. For example, when a firm’s precautionary effort can affect the entire distribution of possible damage (not just the probability of an accident), then threshold liability is shown to minimize the government enforcement costs required to achieve desired precautionary effort, even absent any motive to sort injurers.

This paper’s integration of enforcement concerns (see Polinsky and Shavell 2000; Garoupa 1997) and appeal screening opportunities (see e.g. Bebchuk 1984; Grossman and Katz 1983; Png 1987; Garvie and Lipman 2000; Daughety and Reinganum 2000) yields another contrast to received theory in law enforcement. When there is an endogenous harm-specific probability of apprehension (or prosecution), costs of achieving deterrence—in the form of higher apprehension rates—typically imply optimal under-deterrence of accidents (e.g. Kaplow and Shavell 1994). Alternatively, when the probability of apprehension is exogenous to the specific harm that an accident may cause, prior work argues that accident penalties should be adjusted to account exactly for all accident damages, including costs of enforcement (Polinsky and Shavell 1992). Here, in contrast, the firms that are subject to costly enforcement— the low-damage firms—are optimally over-deterred in order to mitigate costs of appeal screening.

The use of fines and appeals to screen injurers is similar to the suit, settlement and trial literature. Indeed, the fine modelled in this paper can be interpreted as a government offer to ‘settle’ a case. Grossman and Katz (1983), for example, identify the salutary role of plea bargains in screening between guilty (settling) and innocent (appealing) defendants when a court proceeding is costless and informative, but not perfectly informative. Here, however, the court can optimally discriminate between defendants, although the costs of their use motivate an alternative sorting mechanism, i.e. the payment of regulatory fines; courts thus play a valuable role in information discovery, while the payment of fines serves to save enforcement costs.

The remainder of the paper is organized as follows. Section I presents the paper’s core model. Section II characterizes optimal government enforcement policy when there are no appeals and appeal-screening, respectively, describing circumstances under which the latter outcome is welfare-enhancing. Section III considers two key model variants, one allowing for regulators to freely observe a coarse signal of damage, and the other allowing precautionary effort to affect the entire distribution of accident harm. Section IV concludes. An Appendix contains proofs.
I. THE MODEL

Consider a set of risk neutral firms that can engage in activities that can lead to damaging accidents. Firms are of two types, ‘high-damage’ and ‘low-damage’, denoted by $t = h$ and $t = l$, respectively. Each firm has private information about its type. The proportion of $h$ types in the population of potentially injuring firms is $q \in (0,1)$. Firms exercise care that reduces the likelihood of an accident. The level of care is $x$, and the probability of an accident is $p(x, t)$, where $t = h$ or $l$. $p_x(\cdot) < 0$ and $p_{xx}(\cdot) > 0$ (care reduces accident risk at a decreasing rate). Without paying any accident-related costs, a firm’s expected profits are $\pi(x, t)$, where $p_{x} > 0$ and $p_{xx} < 0$ (care is increasingly costly).

When an accident occurs, damages to the general public are $d$, where $d$ is the realization of a random variable that has the positive and twice differentiable density function $g(d; t)$ on the support $[d; d^2 / C]$. $h$ type firms have worse damage distributions in the sense of the monotone likelihood ratio property (MLRP):

$$\frac{\partial}{\partial d} \left( \frac{g(d; h)}{g(d; l)} \right) > 0 \quad \forall d.$$  

The MLRP is a somewhat stronger condition than first-order stochastic dominance (FOSD) of $h$’s damage distribution, implying that expected accident damages are higher for $h$ firms than for $l$ firms, $E(d; h) > E(d; l)$. In a ‘first-best’ world, firms can be costlessly assessed damages from their accidents. Firms then select their care levels to solve the following problem:

$$\max_{x \in X} \left\{ \pi(x, t) - p(x, t)E(d; t) \right\}, t \in \{h, l\},$$

where $X$ is a non-empty and compact feasible set. The first-best care levels that solve (2) will be denoted by $x^*(t)$ for $t = h, l$.

In our model, however, the revelation of actual damages $d$ requires a costly court proceeding, without which only a fixed (non-damage-contingent) fine can be levied on an injuring firm. The government regulator and an injuring firm thus engage in the following post-accident game. First, the regulator offers to ‘settle the case’ with the firm’s payment of a fine $f$. Second, the injuring firm either pays the fine (in which case the game ends) or appeals to the court. Third, if the firm appeals, then the regulator either prosecutes or drops the case. If the case is dropped, the firm pays nothing. The relative frequency with which the government prosecutes is denoted by $r \in [0,1]$. Finally, the case goes to court if an appeal is prosecuted. In the court proceeding, the firm and the government bear legal costs of $c_a > 0$ (‘$a$’ for ‘appeal’) and $c_g > 0$ (‘$g$’ for ‘government’) respectively; in addition, the court observes $d$ and assesses liability $l(d)$. Absent a court proceeding, the government regulator knows only that the injuring firm has caused an accident. In these initial stages, the injuring firm is also unaware of the actual damage realization $d$, although (unlike the government) it knows its own ex ante damage distribution.

The government’s policy consists of the fine level, $f$, the prosecution rate, $r$, and the legal rule, $l(d)$. A benevolent social planner selects these policies ex ante (before care levels are determined and accidents have occurred) in order to maximize social welfare. The legal rule, $l(d)$, is constrained so that (i) accidents are not rewarded in court (i.e. $l(d) \geq 0$); (ii) higher damages are not
rewarded with lower liability (i.e. \(l(d)\) is non-decreasing); and (iii) payments do not exceed a firm’s available assets \(y\) (i.e. \(l(d) \leq y\)).

This model is designed to capture the following salient features of the accident regulation process. First, the court process is a costly mechanism for discovering truth—or a valuable signal of truth—about ex post accident damages. Second, the payment of fines (stage 2) provides a mechanism for avoiding the costly court process. Third, the choice between fines and appeal can serve as a self-selection device whereby high and low-damage firms can be separated and thereby treated differently. In the absence of costly information discovery, this separation would not be necessary because, using a strict liability rule, \(l(d) = d\), all firms could be costlessly made to choose first-best care levels. Fourth, self-selection is valuable because the firms have different damage distributions and different optimal levels of care. Absent heterogeneity, a simple fine could be used to elicit the optimal level of care, without a costly resort to the court. Finally, the government designs its policies to minimize both accident costs and legal/enforcement costs.

In principle, there are four possible cases to consider: (1) neither \(l\)-type nor \(h\)-type firms appeal; (2) \(l\)-type firms appeal and \(h\)-type firms pay the fine; (3) both \(l\)-type and \(h\)-type firms appeal; (4) \(h\)-type firms appeal and \(l\)-type firms pay the fine. In what follows, we first analyse the no-appeal case (1), followed by the key ‘appeal screening’ case (2). Finally, cases (3) and (4) are ruled out as inefficient and impossible, respectively.

II. ANALYSIS

Case (1): No appeals

Without appeals, the only relevant government policy parameter is the fine \(f\). This parameter is selected to maximize social welfare as follows:

\[
\max_f q\{\pi(x(f), h) - p(x(f), h)E(d;h)\}
\]

\[
+ (1 - q)\{\pi(x(f), l) - p(x(f), l)E(d;l)\}
\]

where

\[x(f, t) = \text{No appeal x Choice}\]

\[= \arg\max (\pi(x, t) - p(x, tf)).\]

Given the fine \(f\), injuring firms choose the care levels \(x(f, t), t = h, l\). For each firm, this \(x\) choice yields social surplus of \{\(\pi(x, t) - p(x, tf)E(d;t)\}\}. The government chooses \(f\) so that total (per firm) expected social surplus is maximized.

**Proposition 1.** In a solution to problem (3): (a) \(E(d;\bar{l}) < f < E(d;h)\); (b) \(h\) exerts less than first-best care: \(x_h < \arg\max \{\pi(x, h) - p(x, h)E(d;h)\} = x^*(h)\) = first-best \(x\)-choice for \(h\) firm; (c) \(l\) exerts more than first-best care: \(x_l < \arg\max \{\pi(x, l) - p(x, l)E(d;l)\} = x^*(l)\).
In order to balance the efficiency costs of departing from first-best care levels, the government sets \( f \) between the two first-best fines (\( E(d;l) \) and \( E(d;h) \)), thus yielding a levy that is ‘too high’ for the low-damage firms and ‘too low’ for the high-damage firms.

**Case (2): Appeal-screening**

The efficiency motive for appeal screening is to confront the different firms with different average penalties, and thereby to reduce the extent of over and under-deterrence to which they are otherwise exposed; the cost of this differentiation is that court costs are incurred.

Formally, in case (2), the \( h \)-type firms face a fine of \( f \) (because they do not appeal) and the \( l \)-type firms, which appeal, face the accident penalty, \( \rho(E(l(d);l) + c_a) \), i.e. the probability that the appeal is prosecuted, times the sum of the \( l \) appellant’s expected liability and court cost. Assuming that it wishes to support appeal-screening outcomes, the government’s policy choice problem is:

\[
\max_{\{f, \rho, l(d)\}} q\{\pi(x(f, h), h) - p(x(f, h), h)E(d;h)\}
\]

\[
+ (1 - q)\{\pi(x(P(\{l(d)\}, \rho);l), l) - p(x(P(\{l(d)\}, \rho);l), l)[E(d;l) + \rho c]\}
\]

subject to

(a) \( h \) selection (no appeal): \( f \leq \rho(E(l(d);h) + c_a) = \text{cost to } h \text{ of appeal}\);
(b) \( l \) selection (appeal): \( f \geq P(\{l(d)\}, \rho;l) \equiv \rho(E(l(d);l) + c_a) = \text{cost to } l \text{ of appeal}\);
(c) feasibility of \( l \): \( 0 \leq l \leq y \quad \forall \; d \);
(d) feasibility of \( (f, \rho) \): \( 0 \leq f \leq y + c_a, 0 \leq \rho \leq 1 \).

Problem (5) states that the government wishes to maximize the average social surplus subject to (i) appeal-screening, (ii) attendant care choices, \( x(f, h) \) and \( x(P(\cdot), l) \), respectively, and (iii) the feasibility restrictions (c) and (d). Let \( (f^*, \rho^*, l^*(d)) \) denote a solution to this problem. To avoid degenerate outcomes, we assume:

**Assumption 1.** The optimal prosecution rate \( \rho \) is positive, \( \rho^* > 0 \).

Then a necessary property of an optimum is that the legal rule \( l^*(d) \) penalizes \( h \)-type firms as much as possible for a given level of penalty to \( l \)-type firms, \( E(l^*(d);l) \). That is, there cannot be another \( k(d) \) function that (i) satisfies the feasibility constraints in (5c); (ii) yields the same expected penalty to \( l \) firms, \( E(l(k(d);l) = E(l^*(d);l) \), and (iii) yields a higher expected penalty to \( h \) firms, \( E(l(k(d);h) > E(l^*(d);h) \). Otherwise, \( l^*(d) \) could be replaced by \( k(d) \) and the \( h \)-selection constraint thereby relaxed. Moreover, relaxation of the latter constraint permits a revision of the legal rule (increasing \( l \) firms’ expected payment, \( E(l(k(d);l)) \) and a corresponding reduction in \( \rho \) (to preserve \( l \)'s expected

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penalty) that preserves care and appeal choices, but lowers legal costs (by lowering \(r\)).

This logic implies that an optimal legal rule must take a threshold form:

\[
(6) \quad l(d) = l_T(d; d_0) = \begin{cases} 
  y & \text{when } d > d_0 \\
  0 & \text{when } d \leq d_0 
\end{cases}
\]

for some critical \(d_0 \in [d, \bar{d}]\). In words, an \(l_T(d; d_0)\) threshold rule assesses maximal liability when damages are high (\(d > d_0\)) and zero liability when damages are low (\(d \leq d_0\)).

Consider a class of feasible ‘non-threshold’ \(l(d)\) functions that yield the same expected penalty to the low damage firms as does a given (arbitrary) threshold rule:

\[
(7) \quad l(d) \neq l_T(d; d_0) \text{ for } d \in D = \text{non-degenerate subset of } \{[d, \bar{d}]\} 
\]

\[
1. \quad \text{for } d_0 \in (d, \bar{d})
\]

The following relationship can now be established.

**Lemma 1.** For any threshold rule, \(l_T(d; d_0)\) : \(d_0 \in (d, \bar{d})\), and any associated non-threshold alternative, \(l_A(d) \in L^A(d_0)\),

\[
E(l_A(d); h) < E(l_T(d; d_0); h).
\]

Intuitively, moving to the threshold rule reduces damage payments in low-damage states, wherein the \(l\)-type firms have relatively greater probability weight, and increases payments in high damage states, wherein the \(h\)-type firms have relatively greater probability weight. Thus, whenever threshold and non-threshold legal rules yield the same expected penalty to \(l\)-type firms, the threshold rule yields a higher expected penalty to \(h\)-type firms.

Lemma 1 implies that only a threshold rule can maximally penalize \(h\)-type firms, the necessary property for an optimum discussed earlier.

**Proposition 2.** A solution to (5) must give \(l(d)\) a threshold (equation (6)) form.

Becker’s (1968) famous argument suggests that legal liability should *always* be set as high as possible (\(l(d) = y\) for all \(d\)) in order to achieve a given accident deterrent \((\rho E(l(l(d)); \cdot))\) at minimum possible cost in prosecution \((\rho c)\). Proposition 2 would be trivial if it reflected another application of this logic. However, in the present context, uniformly maximal liability voids the purpose of the courts—namely, to assess penalties that distinguish between firms with different damage distributions. If liability is uniformly maximal (with \(d_0 = d\)), all firms face the same ‘fine’, \(f = \rho (y + c_0)\); hence, a no-appeal (case (1)) regime can elicit the same care choices (with an identical fine), while saving all enforcement costs.

**Lemma 2.** An appeal-screening (case (2)) optimum can yield higher social welfare than a pure fine (case (1)) optimum only if \(l^*(d) < y\) for some \(d\), a
condition that holds when (a) $E(d;h) - E(d;l)$ is sufficiently large; (b) $g(d;l) - g(d;h)$ is sufficiently large; and/or (c) $c$ is sufficiently small.

Lemma 2 provides a new exception to the Becker argument. For low-damage levels, the sanction is optimally relaxed in order to differentiate between the high-damage firms that have little likelihood of enjoying the reduced sanction, and the low-damage firms that have a substantial likelihood of enjoying the break. In this way, low-damage firms may be afforded a lower average sanction—optimally reflecting the lower cost of their activities—without sacrificing deterrence of the high-damage firms which stick with their fixed fine. The benefits of this differentiation are tied to the extent of difference in damages (condition (a) of Lemma 2) and the extent to which a break in low-damage sanctions can serve to sort the firms (condition (b)). The cost of this differentiation is the need for prosecution (condition (c)).

We now focus on cases in which appeal screening can be socially desirable.

**Assumption 2.** In a solution to (5), liability is not uniformly maximized: $d_0 > d$.

Together, Assumptions 1 and 2 imply that, starting from a solution to problem (5), it is possible marginally to lower the prosecution probability $\rho$, and marginally to lower $d_0$ in tandem (raising liability), so as to preserve the low-damage firms’ expected penalty, $\rho(E(h) + e_0)$. Since such a change saves court costs and preserves firms’ care and appeal choices, it increases welfare so long as $h$ firm choices are also unaffected. Moreover, $h$ firm choices are unaffected if the $h$-selection constraint is slack. Because welfare improvements cannot be possible from an optimum, we have the following lemma.

**Lemma 3.** In a solution to problem (5), (a) the $h$-selection constraint binds and (b) the $l$-appeal constraint is slack.

By exactly the same logic, legal costs can be saved and efficiency increased if the $h$-selection constraint can be relaxed. Moreover, the $h$ firms’ incentive to appeal can be reduced either by lowering the regulatory fine (which has the cost of eliciting lower high-damage injurer care choices) or by raising liability assessments (which has the cost of eliciting higher care choices by the low-damage appellants). From first and second-best benchmarks, the efficiency costs of these penalty changes (in terms of deterrence) are negligible; both changes thus increase efficiency by reducing costs of appeal-screening.

**Proposition 3.** In a solution to problem (5): (a) $h$ firms exert less than first-best care: $x_h < x^*(h)$ = first-best $x$-choice for $h$ firm; and (b) $l$ firms exert more than second-best (and first-best) care:

$$x_l > \arg\max \{\pi(x, l) - p(x, l)[E(d;l) + \rho c]\}$$

= ‘second-best’ $x$-choice for $l$ firm $> x^*(l)$.

Using the courts to sort the injurers, as is done in our appeal screening case (2), need not be efficient if court costs are sufficiently high. However, the following factors favour use of the courts in public enforcement (v. no appeals):

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lower court costs, (ii) greater differences between the damage distributions of low and high-damage injurers, and (iii) higher asset bounds. With greater differences in damage distributions, there are greater benefits of the differential penalties made possible by appeal screening and, in addition, a greater ability of a ‘threshold’ rule to achieve differential penalties. Higher asset levels also yield greater differences in the expected ‘threshold’ liabilities of high and low-damage injurers, thereby permitting appeal-screening and deterrence with a lower probability of costly prosecution. Use of the court process is thus likely to be most socially advantageous when injurers have deep pockets.

Cases (3) and (4): Impossibility and Inefficiency

The MLRP condition (1), together with monotonicity of \( l(d) \), imply the following lemma.

**Lemma 4.** \( E(l(d);l) \leq E(l(d);h) \).

With \( l(d) \) non-decreasing, \( h \) firms have greater probability weight on higher-damage outcomes that are associated with larger payments; hence their expected penalty, \( E(l(d);h) \), is higher than for their \( l \)-firm counterparts. As a result, the appeal mechanism is necessarily more advantageous for the \( l \)-type firms, giving us the following corollary.

**Corollary 1.** Case (4) (\( h \) firms appeal and \( l \) firms do not) is impossible.

Finally, if all firms were to appeal (case (3)), the \( h \) firms could be offered an equivalent regulatory fine that deters their appeal (but does not deter appeal by the \( l \) firms), preserves deterrence, and thus saves court costs.

**Corollary 2:** Case (3) (both \( l \) firms and \( h \) firms appeal) is inefficient.

Case (3) may be interpreted as the outcome of this paper’s game modified either to do away with the fine stage or to replace the court process with the regulator’s observation of \( d \) in stage 1 at the cost \( c_p \) to the injurer and \( c_g \) to the regulator. Corollary 2 states that neither modification is desirable, thus implying that there is value to the separation of the fine and court stages of the game posited here.

III. EXTENSIONS

(a) Rough signal of harm

So far, we have assumed that regulators are completely uninformed about the damage from an accident, and that revelation of damage requires a costly court proceeding. However, in practice, regulators may sometimes obtain a coarse signal of damage without investing any investigatory resources in an enforcement action. Regulatory fines can then be conditioned on this signal. Nevertheless, probabilistic enforcement action—interpreted here as going to
court—may be advantageous, because it yields a complete assessment of true damage and thereby permits a finer damage-contingent penalty schedule.

To explore this possibility, let us suppose that regulators freely observe whether damage is high, \( d \in D_+ \equiv [D, \bar{d}] \), or low, \( d \in D_- \equiv [d, D] \), for some \( D \in (d, \bar{d}) \), although they do not observe the specific realization of damage within these intervals. Regulatory fines are thus a pair \((f_-, f_+)\), with \( f_- \) imposed when \( d \in D_- \), \( f_+ \) imposed when \( d \in D_+ \), and \( f_+ \geq f_- \geq 0 \). (Fines, like liability, cannot fall with damage.)

Define the set of fines that confront a \( t \)-type firm \((t \in \{l, h\})\) with its true expected damage, as follows:

\[
I_t \equiv \{(f_+, f_-) : G(D; t)f_- + (1 - G(D; t))f_+ = E(d; t)\}
\]

By condition (1), low-damage firms have steeper \( I_t \) correspondences (see Figure 1):

\[
\left. \frac{df_+}{df_-} \right|_{I_t} = \frac{-G(D; l)}{1 - G(D; t)} < \frac{-G(D; h)}{1 - G(D; h)} = \left. \frac{df_+}{df_-} \right|_{I_h}.
\]

Moreover, the low-damage firms’ \( I_l \) intersects the 45° line (at \( f_+ = f_- = E(d;l) \)) below the corresponding intersection for high-damage firms (at \( f_+ = f_- = E(d;h) \)). Hence, if the \( f_- \) intercept for \( I_l \) is on or above the corresponding intercept for \( I_h \), \( I_l \) and \( I_h \) intersect at a fine pair that yields both firm types exactly their expected damage. In this case, the damage signal is completely informative in the sense that no further information is of any economic value; that is, appeals to the costly court process are not desirable.

There are three reasons why such first-best regulatory fines may not be feasible: (1) the damage signal may not be perfectly informative; (2) asset
bounds may bind; and (3) there may be three (or more) types of firm, so that our two-partition signal, although perfectly informative for two firm types, is not so for three.18

Beginning with the first of these reasons, note that the intercept is above the intercept if and only if

\[ \frac{1 - G(D; h)}{1 - G(D; l)} \geq \frac{E(d; h)}{E(d; l)} = \int_d^D \frac{(1 - G(d; h))}{dd} \int_d^D \frac{(1 - G(d; l))}{dd}. \]

By equation (1), condition (9) is violated when the signal partition is either sufficiently high or sufficiently low. In the latter (not completely informative) case, we have Figure 1 and the following proposition.

**Proposition 4.** Suppose that regulators obtain a coarse signal of damage that is not completely informative (so that equation (9) is violated). Then, absent any appeals to the courts, an optimal policy of regulatory fines will (a) set \( f^- = 0 \); (b) under-deter high-damage firms (relative to a first-best); and (c) over-deter low-damage firms.

In other words, a no-appeal optimum gives us a threshold rule in fines, although without bankrupting penalties \( (f_+ < y) \); low-damage realizations, \( d \leq D \), are not fined at all, a common practice in environmental law enforcement link (Harrington 1988). To understand this result, note first (referring to Figure 1) that there are three possible regions for the fine pair, \((f_-, f_+)\): region C (where all firms are over-deterred), region A (where all are under-deterred), and region B (where \( l \) types are over-deterred and \( h \) types under-deterred). In region C (or A), welfare can be raised by lowering (raising) fines; hence an optimal pair is in the interior of region B (giving us Proposition 4(b)–(c)). Now consider an arbitrary pair in this region, with \( f_- > 0 \) (contrary to Proposition 4(a)). Then \( f_+ \) can be raised and \( f_- \) lowered so as to preserve the expected fine faced by \( l \)-type firms. By the MLRP condition (1), this change will raise the expected fine faced by the \( h \)-type firms and hence raise welfare by reducing the extent to which \( h \)-firms are under-deterred.

Appeals to a costly court process—and attendant imposition of liability—can also be desirable despite the government’s improved ability to levy damage-contingent regulatory sanctions. For example, consider a threshold legal rule in concert with the regulatory optimum of Proposition 4. Specifically, suppose that the threshold \( d_0 > D \), and the high-damage prosecution rate \( \rho_+ \), are selected to make the \( h \)-type firms indifferent between appeal and no appeal (for \( d \in D_+ \)):19

\[ \rho_+ E(l_T(d; d_0); h) = (1 - G(D; h))f_+ \Rightarrow \rho_+ E(l_T(d; d_0); l) < (1 - G(D; l))f_+. \]

Then \( h \) firms do not appeal, \( l \) firms do, and the extent of \( l \)-firm over-deterrence is reduced. Because \( h \)-firm deterrence does not change, welfare increases provided the cost of the appeals \(-(1 - q)\rho_+ c(1 - G(D; l))p(x_T, l)\)—is sufficiently small.

A second reason why first-best regulatory fines may not work is that the asset limit, \( f_+ \leq y \), may bind (see Figure 2). In this case, we have (due to logic akin to Proposition 4) the following.

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Proposition 5. Suppose that equation (9) is satisfied, but a first-best fine exceeds the asset limit, \( f_+ > y \). Then, absent court appeals, an optimal regulatory policy (a) sets \( f_+ = y \), (b) under-deters high-damage firms, and (c) over-deters low-damage firms.

Appeals can again be desirable. Consider, for example, a threshold legal rule in concert with the no-appeal optimum of Proposition 5. Here, the threshold—and appeals—will be in the low-damage region, \( d_0 < D \). Designing this rule to just preserve high-damage firms’ no-appeal incentive, we will again see low-damage firm appeals, an attendant reduction in low-damage firms’ over-deterrence, and welfare gains—provided that the appeal costs, \((1 - q)p_c G(D;l)p(x_l, l)\), are sufficiently small.

Finally, if there are three (not two) firm types, then the two fines, \((f_-, f_+)\), may not be sufficient to confront all three firm types with the respective average harms caused. Specifically, let us consider the addition of ‘very high’-damage firms, \( t = v \), and rule out the prior two exceptions to first-best regulation.

Assumption 3. (i) The intercept, \( f_+(t) \equiv E(d; t)/(1 - G(D; t)) \), rises with quality \( t \) \((l > h > v)\). Hence the damage signal is perfectly informative for any two firm types. (ii) Asset bounds do not bind, \( f_+(\ell) \leq y \). (iii) Low-damage (\( \ell \)) firms are over-deterred at first-best fines for the two higher-damage (\( h \) and \( v \)) firms.21

The third requirement of Assumption 3 (verified in an example in note 21) implies that a first-best is not possible. For this case, we have Figure 3 and the following implications.

Proposition 6. With three firm types and Assumption 3, an optimal regulatory policy, absent court appeals, under-deters high-damage (\( h \)) firms and over-deters both low (\( l \)) and very high (\( v \))-damage firms.22
To understand Proposition 6, consider Figure 3. It suffices to rule out areas A–F as inefficient. The blank area (where \( l \) and \( v \) are over-deterred and \( h \) is under-deterred) then represents the only remaining possibility for an optimal allocation. Areas C (all under-deterred) and F (all over-deterred) can be ruled out immediately. In area A (\( l \) under-deterred, \( h \) and \( v \) over-deterred), \( f_+ \) can be lowered and \( f_- \) raised to preserve \( l \) firms’ expected sanction, thereby reducing the over-deterrence of \( h \) and \( v \) firms. In area B (\( l \) and \( h \) under-deterred, \( v \) over-deterred), \( f_+ \) can be lowered and \( f_- \) raised to preserve \( h \) firms’ expected sanction, thereby reducing the under-deterrence of \( l \) and the over-deterrence of \( v \). Finally, in areas D and E (where \( l \) is over-deterred and \( v \) is under-deterred), \( f_+ \) can be raised and \( f_- \) lowered to preserve \( h \) firms’ average sanction, thereby reducing the over-deterrence of \( l \) and the under-deterrence of \( v \).

When first-best regulatory fines are not possible (for any of the three reasons explored above), an optimum may involve the higher-damage firms paying regulatory fines, and the lowest-damage (\( l \)) firms appealing to the courts in order to lower their sanction and thereby reduce the extent of their over-deterrence. Appeals may occur (i) when the damage signal is low (\( d < D \), \( f = f_- \)), (ii) when it is high (\( d > D, f = f_+ \)), or (iii) in both cases. In all of these cases, the logic of Proposition 2 applies. Specifically, a modified threshold rule of the following form is necessarily optimal (see Figure 4):

\[
l(d) = l^*_T(d; d_0, d_1, \ell) = \begin{cases} 
  y & \text{when } d > d_1 (> d_0) \\
  l \in [0, y] & \text{for } d \in [d_0, d_1] \\
  0 & \text{for } d < d_0.
\end{cases}
\]

The reason is that any non-threshold rule can be replaced by a modified threshold rule that (1) sets \( \ell = l(D) \), and (2) sets \( d_0 \) and \( d_1 \) to preserve \( l \) firms’ conditional expected liability in regions \( D_- \) and \( D_+ \), respectively. The revised legal rule relaxes the higher-damage firms’ selection (no appeal) constraints,

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and preserves all firms’ precautionary incentives. Relaxation of the selection constraints in turn permits the thresholds \(d_0\) and \(d_1\) to be lowered—so as to preserve the \(l\) firms’ deterrence; this can be done without violating the higher-damage firms’ incentive to pay the relevant fine, thereby saving enforcement costs.

**Proposition 7.** Suppose that, in an optimum, the lowest-damage firms appeal to the courts (for \(d \leq D/C_0\); \(d \leq D_+\), or both) and higher-damage firms do not. Then: (a) an optimal legal rule takes the modified threshold (equation (6)) form; (b) if appeals occur only when \(d \leq D/C_0\), then \(l = y\) and \(d_1 = D\); and (c) if appeals occur only when \(d \leq D_+\), then \(l = f\), and \(d_0 = d\).

If appeals occur only in one region of damage \((D_-\) or \(D_+)\), then optimal liability takes a ‘single-step’ threshold form. When appeals occur only in \(D_-\), then high-damage firms’ incentive to pay regulatory fines is maximized by setting \(l\) maximally (to \(y\)), thus giving us a pure (equation (6)) threshold rule. When appeals occur only in \(D_+,\) then these no-appeal incentives are maximized by minimizing \(l\) subject to no appeal occurring in region \(D_-\). This is done by setting liability uniformly equal to the fine \(f\) for the low-damage realizations \((d \in D_-)\), and costlessly committing to certain prosecution for any low damage appeals (a costless commitment because no appeals occur for \(d \in D_+\)). Hence, we again have a ‘pure’ threshold rule, except that liability for low damages is positive, not zero.

(b) **Optimal threshold penalties in a pure incentive model**

In the foregoing analysis, a threshold penalty function is motivated as an optimal mechanism to minimize costs of screening heterogeneous injurers. A
threshold rule also arises as an optimal mechanism in a pure incentive model with homogeneous injurers and enforcement costs.

So far we have considered a care choice that affects only the probability of an accident, but not the level of realized damage. Suppose instead that precautionary effort affects the entire distribution of damage. Formally, let damage be distributed according to the density \( g(d; x) \) on the support \([0, d]\), where higher care leads to lower damages in the sense of the MLRP:\(^{24}\)

\[
(l') \quad \frac{\partial}{\partial d} \left( \frac{g_x(d; x)}{g(d; x)} \right) < 0 \quad \forall d.
\]

Let us also suppose that injurers are homogeneous and have expected profit, absent accident-related costs, equal to \( \pi(x) \) (with \( \pi' < 0 \) and \( \pi'' < 0 \), as before).

Damages are costly for the government to assess, whether because the discovery of damage requires costly monitoring (the assumption in the enforcement literature and this paper) or because there are fixed costs associated with the assessment of any penalties (arising from legal disputes, for example). Specifically, an accident penalty of \( l(d) \in [0, y] \) is levied on a firm with the probability \( \rho \). \( l(d) \) is assumed to be monotone non-decreasing and piecewise differentiable. The government bears the fixed cost \( c \) whenever the \( l(d) \) penalty is imposed. This structure will be called model 2.

A firm’s care choice problem is then

\[
(11) \quad \max_{x \in X} \{ \pi(x) - \rho E(l(d); x) \},
\]

which yields the unique care choice \( x(\{l(d)\}, \rho) \).\(^{25}\) The social planner’s problem thus becomes

\[
(12) \quad \max_{\{l(d)\}, \rho} \{ \pi(x) - E(d; x) - \rho c \},
\]

s.t \( (i) \) \( x = x(\{l(d)\}, \rho) \), and \( (ii) \) \( l(d) \in [0, y] \).

In Model 2, regulatory fines that are completely independent of damage provide no incentive at all for precaution. Hence the penalty function described here is either court-imposed (the interpretation from our earlier model) or based upon a costly regulatory investigation of damage. Moreover, it is the costs of imposing these penalties—the enforcement costs of interest in this paper—that motivate a threshold penalty function.\(^{26}\)

Lemma 5. For fixed \( \rho > 0 \), consider (i) an arbitrary non-threshold rule \( l_0(d) \) that yields care \( x_0 = x(\{l_0(d)\}, \rho) \), and (ii) a threshold (equation (6)) alternative that yields the same expected penalty at care level \( x_0 \):

\[
E\{l_T(d;d_0); x_0\} = E\{l_0(d); x_0\}.
\]

The threshold alternative yields higher precautionary effort: \( x(\{l_T(d;d_0)\}, \rho) > x_0 \).

Intuitively, moving to a threshold rule raises penalties in high-damage states and lowers penalties in low-damage states. Moreover, higher care levels reduce the probability of high-damage states and raise the probability of low-damage states. Hence threshold liability maximizes a firm’s benefit of care by maximizing payments in those states for which higher care is likelihood-
reducing and minimizing payments in states for which higher care is likelihood-increasing.

The optimality of a threshold rule is now clear. Provided there is positive deterrence in an optimum \((\rho > 0)\), the social planner can (i) replace any non-threshold penalty function with the expected-penalty-preserving threshold rule of Lemma 5 (which raises care); (ii) lower the prosecution rate in tandem (which lowers care); and (iii) thereby preserve precautionary incentives. Social welfare is then raised, by virtue of the saved enforcement costs.

**Proposition 8.** In Model 2, an optimal penalty function takes the threshold (equation (6)) form, with \(d_0\) satisfying \(g_x(d_0,x) = 0\) at \(x = x(\{l_T(d_0)\}, \rho)\).\(^{27}\)

The incentive motive for threshold liability does not change with heterogeneous injurers that have different costs of care. However, in such a setting the question of whether one can optimally sort high and low-cost injurers using fines and appeals arises, as before. For example, suppose that regulators receive a two-partition signal of damage as in Section III(a) above. Then, absent appeals, a firm’s precautionary effort is a function of the difference between high and low-damage fines, \((f_+ - f_-)\). So long as there are any costs of penalty assessment, however small, an optimal regime of fines thus stipulates a zero levy when damages are low \((f_- = 0)\), giving us a threshold rule in fines (albeit without bankrupting penalties). An optimal high-damage fine \((f_+)\) weighs the relative incentive benefits for low and high-cost firms. Under certain conditions, the optimal fine will over-deter low-cost firms and under-deter high-cost firms.\(^{28}\) Low-cost firm appeals to a court-imposed threshold rule (prosecuted probabilistically) may again serve to mitigate these firms’ over-deterrence at the cost of prosecution and damage discovery.

**IV. CONCLUDING REMARKS**

In practice, regulatory sanctions are often subject to legal challenge, occupying substantial regulatory and private resources (Sablatura 1995; Shortlidge 2003). In addition, legal rules sometimes distinguish between different damage realizations in ways similar to the threshold rule characterized. For example, some State pollution statutes in the USA punish ‘egregious’ violations with asset-liquidating penalties and exempt ‘non-egregious’ violations from sanctions (Cory 1997).

In some cases, however, this paper suggests scope for reform of regulatory/legal practice that would economize on use of the costly court process. In US Superfund enforcement, for example, litigation reportedly consumes a third of all related private expenditures (Sablatura 1995). By imposing joint and several liability, Superfund can lead to asset-liquidating penalties akin to those prescribed by a threshold rule. However, unlike a threshold rule, it does not afford liability exemptions to firms that cause little realized harm.\(^{29}\) Moreover, in some regions of the United States the Environmental Protection Agency (EPA) has pursued a prosecutorial/liability remedy as a general rule of Superfund enforcement; other regions have more vigorously sought settlements.
that save vast legal costs (Church and Nakamura 1993). In recent years, the EPA has taken steps to encourage settlement by small (‘de minimus’) potentially responsible parties (PRPs), including providing funding for ‘orphan shares’ of insolvent PRPs, promoting mixed (federal and private) funding of some cleanups, clarifying and streamlining procedures for de minimus settlements, and issuing guidance to regional offices that encourage such settlements (Stoody 2001; CEQ 1993). Despite these reforms, Superfund’s severe liability rules—and the ability of large PRPs to recover damages from other parties—has promoted defensive litigation and caught many apparently innocent parties in Superfund’s web (Stoody 2001; Carson 1996). In some regions, it also appears that little has been done to spur settlement among non-de minimus PRPs. This analysis suggests that efficiency may be enhanced by (i) providing liability exemptions to firms found to be relatively small (low-damage) contributors to a contaminated site, as with a threshold rule, and (ii) removing other impediments to settlement by, for example, encouraging ‘buyouts’ that limit a PRP’s future liability and increased use of an ‘accommodation’ (v. ‘prosecution’) strategy in dealing with PRPs (Church and Nakamura 1993).

On a conceptual level, there are a number of possible generalizations to this analysis that may merit attention. I close with comments on a few of them.

A central premise in this paper is that, absent a court proceeding, a firm does not observe the realized level of harm that its accident has caused, although it knows the nature of its activities and thus the distribution of possible harm. For chemical leaks and other toxic pollutant releases, specific economic damages from an accident are complex to calculate and thus are likely to be unobservable to firms—or the government—absent a costly inquiry, interpreted here to be the domain of a court process. However, in other cases there may be an imperfect signal of damage; or the firm may actually observe realized harm (and not the government).

When regulators obtain an informative signal of harm, the merit of this paper’s logic hinges upon the strength of the signal; if it is very informative, signal-contingent regulatory fines may costlessly approximate strict liability and thereby void any economic motive for costly court challenges. Nevertheless, as discussed in Section III, the conclusions of this analysis are, in many cases, robust to imperfectly informative signals of harm.

If firms observe their damages, there will no longer be any appeal-screening motive for use of the courts; regardless of its type, a firm will appeal to the court (or not) depending only upon whether the resulting known liability (times the known prosecution rate) is lower (or higher) than the regulatory fine. However, the economic logic of both threshold liability and the separation of regulatory fines from court appeals can be shown to persist in general. Costs of assessing liability imply that high-damage injurers will be under-detained in an optimum; a threshold rule is thus optimal because, preserving the expected penalty to low-damage injurers, it minimizes the extent to which high damage injurers are under-detained. Given a threshold rule, there is no economic motive for injurers that have high realized damages—those that would face a maximal sanction under the threshold rule—to appeal to the courts; deterring such appeals by setting the regulatory fine equal to the average sanction that
would otherwise be levied (in the paper’s notation, \( f = \rho y \)) preserves deterrence, but saves court costs.

The merits of threshold liability, in view of costly enforcement, similarly generalize to cases in which the distribution of harm is not exogenous, but rather is determined by precautionary measures undertaken to limit damage (e.g. careful monitoring of production activities, so that accidental chemical releases are quickly detected and stopped). As was shown in Section III, when damages are costly to assess and the full distribution of damages is determined by \( ex \ ante \) care (as in Lewis and Sappington 1999), a threshold rule provides a maximal incentive for the exertion of care and thereby minimizes the requisite probability of penalty assessment needed to achieve a given deterrent to damage creation. When firms have heterogeneous (and hidden) costs of damage-reducing care, an appeal-screening mechanism may be able to sort the injurers optimally, provided regulatory fines can be conditioned on a partially informative signal of harm. Absent such a signal, payment of fines—by completely decoupling sanctions from damage—will eliminate incentives to limit harm. With such a signal, optimal regulatory fines will take a threshold form, completely exempting firms with low-damage realizations from sanction—a common practice in environmental law enforcement (Harrington 1988). Moreover, as stressed throughout the paper, it may be optimal for low-cost (high-precaution) firms to appeal to a court process and thereby mitigate the excessive deterrence to which they may otherwise be subject.

A potential criticism of the threshold rule is that it may provide an incentive for a firm, \( ex \ post \), to limit its accident damage to a level below the threshold and thereby escape liability entirely. This might be done by remediation effort or might take a more pernicious form, with effort to split up the damage between multiple sites. Implicitly, we have assumed that the courts can see through such efforts. For example, the courts can surely aggregate a ‘split-up’ hazard and thereby vitiate any motive for such costly behaviour. However, in this paper I have not considered how regulatory law enforcement might provide proper incentives for damage remediation (Heyes 1996; Innes 1999b), or avoid excessive firm investment in pre-appeal information about damages, or account for imperfect damage assessments (Kaplow and Shavell 1996), all of which may merit further study.

A final caveat is that the extreme bankrupting penalties that are advocated in this paper are rarely observed. Other considerations would optimally lead to less extreme penalties. For example, deadweight costs of financial distress and bankruptcy (costs that have been extensively studied in the finance literature) and injurer abilities to take costly measures that limit their assets (measures that, Ringleb and Wiggins 1990 argue, have had a pervasive effect in the United States) both favour liability limits that are set below asset bounds. Constitutional strictures against confiscation of property may likewise favour alternative liability limits linked to either \( ex \ post \) damages or measures of expected damages. The logic of this analysis—and of the optimal threshold liability rule characterized here—is robust to these alternative liability bounds, with maximal liability appropriately redefined. However, such alternative bounds will clearly lead to less extreme accident penalties, as will such considerations as risk aversion (see e.g. Polinsky and Shavell 1979) or ‘fairness’ (see e.g. Abraham and Jeffries 1989).

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Proof of Proposition 1

The following properties are easily verified (for $t = l, h$):

(i) $\pi^*(x, t) = p^*(x, t)E(d; t) \geq 0$ when $x \geq x^*(t)$
(ii) $x(f, t) \geq x^*(t)$ when $f \geq E(d; t)$
(iii) $\partial x(f, t)/\partial f > 0$.

Properties (i)--(iii) imply that the derivative of the objective function in (3) (with respect to $f$) is (I) positive when $f \leq E(d; l) < E(d; h)$, and (II) negative when $f \geq E(d; h) > E(d; l)$, which establishes part (a) of the Proposition. Together with part (a), property (ii) implies parts (b) and (c).

Proof of Lemma 1

Define

$$\phi(d) = l_A(d) - l_T(d, d_0) \geq 0 \quad \forall d \leq d_0$$

and note that, since $E(l_A(d); l) = E(l_T(d, d_0); l)$,

$$\int_d^{d_0} \phi(d)g(d; l)dd + \int_d^d \phi(d)g(d; l)dd = 0.$$

Further, define

$$\delta(d_0) = \frac{\phi(d_L)g(d_L; l)}{\int_d^{d_0} \phi(d)g(d; l)dd} = \frac{\phi(d_L)g(d_L; l)}{-\int_d^d \phi(d)g(d; l)dd} \text{ for } d_L \in [d, d_0),$$

where the second equality follows from (A2). By construction $\int_d^{d_0} \delta(d_L)dd_L = 1$. Using this fact and substituting for $\delta(d_L)$ from (A3), $E(\phi(d); h)$ can be written as

$$E(\phi(d); h) = \int_d^{d_0} \phi(d_L)g(d_L; h)dd_L + \int_d^d \phi(d_H)g(d_H; h)dd_H$$

$$= \int_d^{d_0} \delta(d_L) g(d_L; h) \left[ \int_d^{d_0} \phi(d_H)g(d_H; l)dd_H \right] dd_L$$

$$\quad + \int_d^d \phi(d_H)g(d_H; h)dd_H \int_d^{d_0} \delta(d_L)dd_L$$

$$= \int_d^{d_0} \int_d^{d_0} \delta(d_L)\phi(d_H)g(d_H; l)dd_H \left[ g(d_H; h) - g(d_L; h) \right] dd_H \text{dd}_L < 0,$$

where $d_H$ denotes the variable of integration over the interval $[d_0, d]$ and $d_L$ denotes the variable of integration over $[d, d_0]$. The inequality in (A4) follows from $\delta(d_L) \geq 0 \quad \forall d_L < d_0$ (with strict inequality on a set of positive measure in the interval, $[d, d_0)$), $\phi(d_L) \leq 0 \quad \forall d_L > d_0$ (with strict inequality on a set of positive measure in the interval $[d_0, d)$), and the MLRP equation (1) (since $d_H > d_L$). The lemma follows from (A4) and the definition of $\phi(d)$. □

Proof of Proposition 2

Suppose instead that there is a non-threshold solution to (5), $l_A(d)$, with an associated prosecution probability, $p^+ > 0$ (Assumption 1), and fine $f$. Then there is a threshold rule $l_T(d; d_0^*)$ that yields $l$ firms the same expected liability as $l_A(d)$,

$$l_T(d; d_0^*) : E(l_T(d; d_0^*); l) = E(l_A(d); l),$$

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and, by Lemma 1, yields $h$ firms a higher expected liability, $E(l_I(d; d_0^h): h) > E(l_A(d): h)$. Since the $h$-settlement constraint (5b) is satisfied with $l_A(d)$ (by construction), this constraint is strictly slack under $l_I(d; d_0^h)$ (given $\rho_A$ and $f$).

Now consider replacing $l_A(d)$ and $\rho_A$ with a threshold rule $l_I(d; d_0)$ and prosecution probability $\rho$, where (i) $\rho$ and $d_0$ are selected to preserve $\rho(E(l_I): c_a) = \rho_A(E(l_A(d): l) + c_a)$; (ii) $\rho$ is marginally less than $\rho_A$ (which lowers $\rho(E(l_I): c_a)$), and (iii) $d_0$ is marginally less than $d_0^h$ (which raises $\rho(E(l_I): c_a)$). With this replacement, (a) $l$ firms' appeal decision and $x$-choice (under appeal) are not affected; (b) $h$ firms' $x$-choice (under no appeal) is unaffected (since $f$ is unchanged); (c) the $h$-settlement constraint is still satisfied since it was slack under the policy $(f, \rho_A, l_I(d; d_0^h))$; and (d) since $\rho$ is lower than $\rho_A$, legal costs are saved and social surplus is raised. Hence the alternative policy, $(f, \rho, l_I(d; d_0))$, satisfies constraints (5a)–(5d) and yields greater expected social surplus (i.e., a higher value of the objective function in (5)) than $(f, \rho_A, l_A(d))$. The posited optimality of the latter non-threshold policy is thus contradicted. □

**Lemma 2’**

If the following condition holds, then $d_0 > d$ in a solution to problem (5):

\[
(A5) \quad \bar{z}(q^{**} - q^*)(E(d; h) - E(d; l) - c)(E(d; h)q^{**} + E(d; l)(1 - q^{**})) > p(x_l, l)(1 - q)c,
\]

where $\bar{z} = q z_h + (1 - q) z_l < 0$

\[
z_l = p_x(x_l, t) / (\pi_{xx}(x_l, t) - p_{xx}(x_l, t)\rho(y + c_a)) < 0
\]

\[
x_l = x(\rho(y + c_a), t)
\]

\[
q^* = q z_h / \bar{z} \in (0, 1)
\]

\[
q^{**} = g(d; h)q^* / (g(d; h)q^* + g(d; l)(1 - q^*)) \in (0, 1)
\]

\[
(q^{**} - q^*) = - q^*(1 - q^*)\Delta(g(d; l) - q^*\Delta) < 0
\]

\[
\Delta = g(d; l) - g(d; h) > 0
\]

and we make the following **Regularity Assumption**: with $d_0 = d$, social welfare ($W$ in (A6)) is locally concave in $\rho$ at any stationary point. [Note on Regularity Assumption. At an optimum, local concavity follows from second-order necessary conditions. However, the regularity restriction imposes local concavity at other possible stationary points, ensuring that there is a unique globally optimal stationary point. Plausible restrictions ensure that this condition is satisfied. For example, if $\pi(x_t, t) = \pi_0(x)\quad \text{and} \quad p(x_t, t) = p_0(x)$, then sufficient for global concavity are (A) $\pi_{xx} \leq 0$ and $p_{xx} \geq 0$, or more weakly, (B) $d \ln \pi_{xx}/d \ln x + (d \ln p_{xx}/d \ln x) - 3(d \ln p_{xx}/d \ln x) \geq 0).$ □

**Proof of Lemma 2’**

Suppose not; suppose $d_0 = d$. Then social welfare can be written

\[
W = q(\pi(x(F_h, h), h) - p(x(F_h, h), h)E(d; h))
\]

\[
+ (1 - q)(\pi(x(F_l, l), l) - p(x(F_l, l), l)(E(d; l) + \rho c)),
\]

where $F_t = \rho[E(l_I(d; d_0):t) + c_a]$ (with both appeal/settlement constraints necessarily binding). To derive a contradiction, it suffices to show that $\partial W/\partial d_0 > 0$ at $d_0 = d$ when $\rho$ is chosen optimally (for $d_0 = d$). To this end, note that, for $d_0 = d$,

\[
\partial W/\partial d_0 = -\rho v\{g(d; h)V_h + g(d; l)V_l\},
\]

\[
\partial W/\partial \rho = (V_h + V_l)(y + c_a) - p(x_l, l)(1 - q)c,
\]

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where (substituting from first-order conditions for \( x(\cdot) \) and \( F_i = \rho(y + c_a) \) when \( d_0 = \underline{d} \)),
\[
\begin{align*}
V_h &= qz^*_h[\rho(y + c_a) - E(d;h)]; \\
V_l &= (1 - q)z_l[\rho(y + c_a) - E(d;l) - \rho c].
\end{align*}
\]

Observe:

Claim 1: (i) If \( V_l \geq 0 \), then \( V_h > 0 \). (ii) If \( V_h \leq 0 \), then \( V_l < 0 \).

Claim 2: There is a \( \rho \in (\rho_2, \rho_1) \): \( \partial W/\partial d_0 = 0 \) at \( \rho = \bar{\rho} \) and \( \partial W/\partial d_0 > 0 \) for all \( \rho > \rho \), where \( \rho_1 = E(d|h)/(y + c_a) > E(d,l)/(y-c^*_h) = \rho_2 \).

Claim 3: If \( \partial W/\partial \rho > 0 \) at \( \rho = \rho \) (and \( d_0 = \underline{d} \)), then any optimal prosecution rate, for \( d_0 = \underline{d} \), is above \( \rho \).

Proof of Claim 1. From the definitions of \( V_r \),
\[
V_l \geq 0 \Rightarrow \rho(y + c_a) \leq E(d;l) + \rho c < E(d;h) \Rightarrow V_h > 0.
\]

Similarly,
\[
V_h \leq 0 \Rightarrow \rho(y + c_a) \geq E(d;h) > E(d;l) + \rho c \Rightarrow V_l < 0.
\]

Proof of Claim 2. First, note that \( \rho_1 > \rho_2 \) follows from \( y \geq E(d|h) > E(d;l) + c \). Second, because \( V_h \leq 0 \) when \( \rho \geq \rho_1 \), \( \partial W/\partial d_0 > 0 \) when \( \rho \geq \rho_1 \) (by Claim 1(ii) and equation (A7)). Likewise, with \( V_l = 0 \) at \( \rho = \rho_2 \), \( \partial W/\partial d_0 < 0 \) at \( \rho = \rho_2 \) (by Claim 1(i) and (A7)). Claim 2 thus follows from continuity of \( \partial W/\partial d_0 \) in \( \rho \) and the Intermediate Value Theorem.

Proof of Claim 3: This follows from the above Regularity Assumption.

By Claims 2 and 3, Lemma 2’ will follow if \( \partial W/\partial \rho > 0 \) at \( \rho = \rho \) when condition (A5) holds. From the definition of \( \rho \) (in Claim 2) and equation (A7), the following holds at \( \rho = \rho \):\n
\[
\rho(y + c_a) = E(d|h)q^{**} + (E(d;l) + \rho c)(1 - q^{**}).
\]

Rewriting \( \partial W/\partial \rho \) in (A8) and substituting from (A10),
\[
\rho(\partial W/\partial \rho)_{\rho = \bar{\rho}} = z\{\rho(y + c_a) - q^*E(d;h) - (1 - q^*) (E(d;l) + \rho c)\} \rho(y + c_a)
\]
\[
- p(x_l, l)(1 - q) \rho c
\]
\[
= z(q^{**} - q^*) (E(d;h) - E(d;l) - \rho c) \rho E(d;h) q^{**}
\]
\[
+ (E(d;l) + \rho c)(1 - q^{**}) - p(x_l, l)(1 - q) \rho c.
\]

Noting that condition (A5) implies that the right-hand side of (A11) is positive completes the proof. \( \square \)

Proof of Lemma 3

(i) If the \( h \)-settlement constraint is slack, then the policy change proposed in the text (marginally lowering \( \rho \) and \( d_0 \) so as to preserve the \( l \) firms’ expected liability \( \rho(E(l;l) + c_a) \)) will not affect \( h \) firm choices, and hence will raise welfare.

(ii) By Proposition 2 and \( d_0 > \underline{d} \) (Assumption 2), we have
\[
E(l(d);h) = y(1 - G(d_0;h)) > y(1 - G(d_0;l)) = E(l(d);l),
\]
where \( G(d):t = \int_0^d g(d;t) \, dd \) and the inequality follows from the MLRP equation (1) (which, as noted earlier, implies the FOSD condition, \( G(d) > G(d;h) \) \( \forall d \in (d, \bar{d}) \)). By (A12), \( \rho(E(l(d);d) + c_d) < \rho(E(l(d);h) + c_d) \). Thus, with a binding \( h \)-settlement constraint, \( \rho(E(l(d);h) + c_d) = f \), the \( l \)-appeal constraint is slack. □.

Proof of Proposition 3

First, note the following necessary properties of a solution to (5): (1) \( l(d) = lT(d;d_0) \) for \( d_0 \in (d, \bar{d}) \) (by Proposition 2 and Assumption 2).

(2) \( f > 0 \) and \( d_0 < \bar{d} \). (In order for the \( l \)-appeal constraint (5b) to hold, \( f = 0 \) if and only if \( d_0 = d \). Hence, if \( f = 0 \) or \( d_0 = d \), (5b) holds with equality, contradicting Lemma 3(b).)

Given Lemma 3(b), Assumptions 1–2, and these two necessary properties of an optimum, the Lagrangean function for problem (5) can be written as

\[
J(f, \rho, d_0) = q\{\pi(x_b, h) - p(x, h)E(d;h)\} + (1 - q)\{\pi(x, l) - p(x, l)[E(d;l) + \rho c]\} + \lambda_H\{\rho(E[l(d);h] + c_d) - f\} + \lambda_F(y + c_a - f) + \lambda_p(l - p),
\]

where \( x_b = x(f, h), x_T = x(P([lT(d;d_0);\rho, l,l]), \) and \( (\lambda_H, \lambda_F, \lambda_p) \) are Lagrange multipliers for the relevant constraints in (5a) and (5d). Differentiating \( J \) with respect to \( f \) and \( d_0 \), setting these derivatives equal to zero and rearranging terms gives the following necessary conditions for an optimum:

\[
\frac{\partial J}{\partial f} = 0 \Rightarrow \pi_x(x_b, h) - p_x(x_b, h)E(d;h) = (\lambda_H + \lambda_F)/[q(\partial x(f, h)/\partial f)] > 0 \tag{A14}
\]

\[
\frac{\partial J}{\partial d_0} = 0 \Rightarrow \pi_x(x, l) - p_x(x, l)[E(d;l) + \rho c] = -\lambda_H p \frac{\partial E(lT(d;d_0);h)}{\partial d_0} \left[ (1 - q) \frac{\partial x(P([lT(d;d_0);\rho, l,l])}{\partial d_0} \right] < 0 \tag{A15}
\]

The inequality in (A14) follows from \( \lambda_H > 0 \) (Lemma 3(a)), \( \lambda_F > 0 \) and \( \partial x(f, H)/\partial f > 0 \). The inequality in (A15) follows from \( \lambda_H > 0 \), \( E(lT(d;d_0);h)/\partial d_0 = -yg(d_0;h) < 0 \), and

\[
\frac{\partial x(P([lT(d;d_0);\rho, l,l])}{\partial d_0} = -\frac{\partial x(P, l)}{\partial f} pyg(d_0;l) < 0. \tag{A16}
\]

Given concavity of the functions, [\( \pi(x, l) - p(x, l)] \), (A14) implies that \( x_b < x(E(d;h);h) = x^*(h) \), and (A15) implies that \( x_l > x([E(d;l) + \rho c], l) = x^{**}(l) \). □

Proof of Lemma 4

Assume \( l(d) \) is piecewise differentiable. (The more general proof is available from the author.) We then have, after integrating by parts,

\[
E[l(d);h] - E[l(d);l] = \int_d^\bar{d} f(d)[G(d;l)G(d;h)] \, dd \geq 0,
\]

where the inequality follows from equation (1) and monotonicity of \( l(d) \) (\( f' \geq 0 \)). □

Proof of Corollary 1

In case (4), \( l \) firms choose to settle and \( h \) firms choose to appeal. Together, these two choices require

\[
A20 \quad f \leq \rho(E(l(d);l) + c_o) \leq \rho(E(l(d);h) + c_o) < f.
\]
The last inequality in (A20) follows from Lemma 4 and implies the contradiction, $f < f$. □

Proof of Corollary 2

Suppose the government sets $(f, \rho, l(d))$ to elicit case (3). Now consider replacing $f$ with $f' = \rho (E(l; h) + c_o)$. Under the policy, $(f', \rho, l(d))$, $h$ firms do not appeal and choose the same care as under $(f, \rho, l(d))$. Moreover, under $(f', \rho, l(d))$, $l$ firms either do not appeal (if $E(l; l) = E(l; h)$), or appeal (if $E(l; l) < E(l; h)$); in either case, the $l$ firms’ care does not change. In sum, the policy $(f', \rho, l(d))$ leads to the same care choices as does $(f, \rho, l(d))$, but to lower legal costs. Hence $(f, \rho, l(d))$ is not efficient. □

Proof of Lemma 5

By arguments parallel to the proof of Lemma 1 above, we have:

\[
\frac{\partial E\{I_r(d; d_0); x_0\}}{\partial x} < \frac{\partial E\{I_0(d); x_0\}}{\partial x}.
\]

Lemma 5 follows directly from (A21) and concavity (in $x$) of the objective function in equation (11). □

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NOTES

1. Shavell (1995) studies a different motive for court appeals—correcting erroneous rulings. In a regulatory context, Garvie and Lipman (2000) identify the potential role of court challenges in sorting injurers; however, in contrast to the present inquiry, they focus on challenges to regulations, rather than the on assignment of liability, and they also treat the prospect (and consequences) of court challenges as exogenous. Rasmussen (1995) also finds that sharp (discontinuous) changes in liability, as harm changes, can be optimal. In a model with non-stochastic (and known) harm that can vary across defendants, Rasmussen characterizes harm-contingent penalties that trade off benefits in deterrence against costs of false convictions.

2. Many other exceptions to the Becker (1968) prescription have been developed in prior work (see Bebchuk and Kaplow (1992) for a survey), including marginal deterrence (Mookherjee and Png 1994), heterogeneous apprehension risk (Bebchuk and Kaplow 1992), remediation (Heyes 1996), heterogeneous wealth, avoidance and bankruptcy costs.

3. This literature abstracts from concerns for precautionary incentives and liability design that occupy the present paper. An exception is Spier (1994), who studies how to design a liability rule that balances the benefits of efficient precaution and the benefits of cost-saving settlements, but does not consider the injurer heterogeneity (and attendant appeal screening) or continuous damages modelled here. In emphasizing court costs, this analysis also builds upon the costly legal process literature (see Polinsky and Rubinfeld 1996). This literature, however, focuses on how to provide optimal incentives for plaintiff (victim) suit for a fixed and homogeneous level of injurer precaution incentive. The present paper, in contrast, is concerned only with injurer incentive issues, as befits the public enforcement problem.


5. Section III considers the situation when regulators receive a coarse signal of damage, so that regulatory fines can depend upon the signal. For conceptual reasons, we first consider a simpler case with no free regulatory information. This case may also be realistic when regulated firms can be distinguished a priori by their class of damage, with some firms in higher ‘damage classes’ than others; for a single damage class (the relevant object of our model), realized damage within the range of possible harms for perceptible accidents may not be distinguishable absent either a costly regulatory inquiry or a court case. For example,
in the case of US Superfund regulation (see Section IV), regulatory authorities have lamented the difficulty of identifying a firm’s contribution to harm (Stoody 2001, p. 28), precisely our premise here.

6. We assume that accidents are costlessly detected by government regulators in order to focus the analysis on the post-accident prosecution of injurers. However, the analysis can be interpreted to allow for imperfect accident detection: if \( p(x; t) \) is the accident probability times the detection probability, and the damage \( d \) is actual damage divided by the detection probability, then this paper characterizes an optimal enforcement policy for a given rate of government detection.

7. The court process reveals \( d \), but not \( x \) or \( t \). That is, care levels are viewed as complex combinations of precautionary measures that render their observation either impracticable or extremely costly (see e.g. Viscusi 1989, or Rose-Ackerman 1991). For simplicity, we assume that the assessment of regulatory fines is costless.

8. It might be argued that, when an accident occurs, the injurer learns something about the damages from that particular accident. Section IV discusses this possibility. However, without a court proceeding and investment of investigative resources, the extent of this information, both as it relates to true underlying damage and even more as it relates to the court’s judgment about this damage, is likely to be limited.

9. This \textit{ex ante} view of policy determination has a couple of motivations. First, in a one-shot game the government may be able to choose between \textit{ex ante} commitment to the parameters, \( (f, p, l(d)) \), and \textit{ex post} discretion. Under \textit{ex post} discretion, sequential rationality on the part of the government would dictate that appeals never be prosecuted, thus saving the court costs and, given that care choices have already been made, having no efficiency effects; discretion will thus lead firms to exert zero care \textit{ex ante}. \textit{Ex ante} commitment, achieved via statutory regulations and budgetary commitments, will therefore permit the achievement of higher welfare than will discretion. Second, even without explicit \textit{ex ante} commitments, the government may seek to achieve such commitments via consistent behaviour over time, based upon which firms make conjectures on their likely fate in the event of an accident.

10. Without loss of generality, \( y \) represents the firm’s asset level after the firm’s legal costs, \( c_\omega \), have been paid. For simplicity, we assume that \( y \) is at least as large as expected damage, i.e. \( y \geq E(dh) \), and is independent of \( x, t \) and \( d \). Beard (1990) considers a model in which a firm’s assets available for damage payment. This paper abstracts from this possibility in order to obtain the \textit{ex post} assets available for damage payment. This paper abstracts from this possibility in order to avoid conditioning liability on assets as a mechanism to condition liability on the firm’s \( t \)-type. This abstraction can be motivated by an ability of firms to pay out profit flows to their owners (e.g. as dividends), leaving the asset level to be driven by operation requirements rather than profits.

11. In this analysis, the asset limit \( y \) operates only to preclude arbitrarily large liability assessments and does not itself imply insufficient incentives for the exercise of care (in contrast to Shavell 1986, Schwartz 1985, Lewis and Sappington 2001 and others). Specifically, \( y \) may (or may not) be larger than the maximal damage, \( d \), thus permitting a strict liability rule, \( l(d) = d \).

12. Without loss of generality here, the legal rule can deter appeals by stipulating \( l(d) = y \) for all \( d \).

13. The government’s choice of \( f \) is subject to the non-negativity and limited liability constraints; \( 0 \leq f \leq y + c_\omega \) = firm assets before payment of the court costs \( c_\omega \). Under premises of this analysis (\( y > E(dh) \)), these constraints do not bind in an optimum.

14. Without loss of generality, we assume that an injurer does not appeal if he/she is indifferent between appeal and no-appeal. Strictly speaking, the \( l \)-agent appeal constraint should thus be written as a strict inequality. However, under plausible conditions this constraint does not bind at an optimum, thus validating the solution to the ‘relaxed’ government choice problem presented here.

15. The fine \( f \) cannot be higher than the firm’s assets before payment of the court costs, \( c_\omega \), i.e. \( f \leq y + c_\omega \).

16. Sufficient for Assumption 1 to be valid are: \( \lim_{x \to 0} -p_x(x; t) \) arbitrarily large, and (ii) bounded limits, \( \lim_{x \to 0} p_x(x; t) \) and \( \lim_{x \to 0} p_x(x; t) \). Absent these regularity restrictions, it is conceivable that the legal costs, \( c_\omega \), are so large that a solution to problem (5) sets the prosecution probability, \( \rho \), equal to zero. However, this degenerate case negates our concern with the legal rule and also implies that case (1) (no appeals with \( f = E(dh) \)) yields higher social welfare than case (2) (appeal-screening with \( \rho = 0 \) and, per constraint (5a), \( f = 0 \)).

17. Consider two asset levels, \( y_0 > y_1 \). If the policy, \( (f_0, l_1(d_0)) \), solves problem (5) with \( y = y_0 \), then the two firm types’ expected accident penalty under appeal are \( p_0(y_0(1 - G(d_0(t))) \) for \( t = l, h \). If the asset level rises to \( y_1 \), the government can lower the prosecution
rate to $p_1 = \rho_0 \nu^0 < \rho_0$, and thereby preserve both firm types’ expected accident penalties and appeal incentives. The higher asset level thus permits achievement of the same appeal-screening outcomes but with lower court costs. Analogous logic implies that appeal screening will be favoured when the difference between the two firm types’ expected threshold liability, $\{G(d_o,l) - G(d_o,h)\}$, is larger.

18. One might argue that the premise of a two-partition signal is as restrictive as two firm types. My point, however, is that a regulatory signal of damage may be more coarse than the distribution of damage types.

19. The implication in (10) follows from Lemma 1.

20. In the high damage region, $d \in D_+$, the government can costlessly commit to a certain probability of prosecution, $\rho_1 = 1$, thus deterring all appeals and avoiding any prosecution.

21. It is possible for two lines to confront any number of different firm types with true expected damage payments, $D \in D(D)$, for which a two-fine optimal solution holds. The solution is unique due to concavity of the objective function, which in turn follows from Assumption 3(iii). The statement of enforcement costs in (12) presumes that the monitoring/penalty assessment cost, $c$, is borne whenever any penalty $(h(d))$ is levied, whether positive or zero. If damage is freely discovered and $c$ represents a penalty assessment cost that is borne only when positive penalties are levied $(h(d)>0)$, then enforcement costs will be lower, equalizing $c$ times $\rho$ (the probability that the penalty $(h(d))$ is levied) times the probability that the penalty is positive $(h(d)>0)$. In this case, the benefits of threshold liability are even greater than described here. The reason is that an expected-penalty preserving threshold rule that preserves $l$ firms’ conditional expected penalties raises higher damage firms’ conditional expected penalties under appeal.

22. The MLRP implies FOSD, $G_x(d,x)>0$ for all $d \in (0,d)$. For simplicity, we also assume that the distribution function is weakly concave, $G_x(x,d)<0$. Results below are robust to relaxation of the concavity premise, but proofs are more complicated.

23. The solution is unique due to concavity of the objective function, which in turn follows from $\pi^*<0$, $f(d) \geq 0$ (monotonicity and piecewise differentiability of $[h(d)]$) and $G_{xx}(d)<0$.

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29. Proposition 8 follows directly from the foregoing arguments. To understand the last clause, recall (from condition (1)’) that $g_x(d,x)>0$ for all $d<d^*$ and $g_x(d,x)<0$ for all $d>d^*$, for some $d^* \in (0,d)$. Hence if $d_0<(<d^*)$ (contrary to Proposition 8), then raising (lowering) $d_0$ to $d^*$ raises marginal incentives to exert care and permits a corresponding reduction in $\rho$ that just preserves care incentives; enforcement costs are thereby saved, and welfare raised.

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32. The following is a sufficient condition for low-cost (high-care) firms to be over-deterred in a natural environment. For example, this condition will hold if $g(\cdot)$ is exponential with $\eta$.

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