Vertical Restraints and Horizontal Control

Robert Innes*

and

Stephen F. Hamilton*

June 23, 2008

Abstract: This paper considers vertical restraints in a multi-product retail setting in which duopoly retailers each sell two manufactured goods. A vertical restraint by a monopoly manufacturer of one good enables horizontal control over the retail pricing of a second, competitively-supplied good. The equilibrium contracts produce symptoms that are consistent with a variety of observed retail practices, including fixed "slotting" fees paid to retailers by competitive suppliers, loss leadership, and predatory accommodation with below-cost manufacturer pricing for dominant brands. Applications are developed for supermarket retailing, where the manufacturer of a national brand seeks to control the retail pricing of a supermarket's private label, and for convenience stores, where a gasoline provider seeks to control the prices of in-store consumption items.

Keywords: contracting, vertical restraints, resale price maintenance, monopolization.

JEL Classifications: L13, L14, L42, D43.

* Robert Innes, Departments of Economics and Agricultural and Resource Economics, University of Arizona, Tucson, AZ 85721. Phone: (520) 621-9741. Email: innes@ag.arizona.edu. Steve Hamilton, Department of Economics, California Polytechnic State University, San Luis Obispo, CA, 93407. Phone: (805) 756-2555. Email: shamilto@calpoly.edu. We thank Bruce Marion and seminar participants at the University of Toulouse for helpful comments on an earlier draft of the paper.

Vertical Restraints and Horizontal Control

1. Introduction

Vertical restraints by manufacturers on the retailers of their products continue to be a source of legal and policy debate. Indeed, in June of 2007, the U.S. Supreme Court reversed course on the legal treatment of resale price maintenance (RPM), overturning its earlier decision on the per se illegality of the practice in favor of a reasoned approach.[1] This ruling is in line with the traditional explanation for vertical restraints that the practice serves to align private incentives between a manufacturer and her retailers in the sale of the manufacturer's good. For instance, intensive price competition among retailers may lead to an inadequate level of pre-sales retail services (Telser, 1960; Mathewson and Winter, 1984; Marvel and McCafferty, 1984; Rey and Tirole, 1986; Klein and Murphy, 1988; Winter, 1993) or promote excessive post-sale quality differentiation (Bolton and Bonanno, 1988). Vertical restraints can correct these distortions, and doing so generally produces efficiency benefits, a point that has been argued by many economists following Bork (1966) and Posner (1976).[2]

This paper considers vertical restraints in a multi-product retail environment. In this setting, we identify more pernicious effects: A restraint on a manufacturer's own good serves as a mechanism to control the retail pricing of another ("rival") manufactured good.

We consider a successive oligopoly market structure with two manufacturing industries, two products and two retailers. In the upstream market, one product is produced by a monopolist and the second product is produced by a "competitive fringe". The two products are bundled together in the downstream retail market in the sense that each retailer carries both goods and each consumer buys both goods from one of the two retailers. Examples of such a vertical

---

[1] See *Dr. Miles Medical Co. v. John D. Park and Sons*, 220 U.S. 373 (1911) and *Leegin Creative Leather Products, Inc. v. PSKS, Inc.,* Slip Op. No. 06–480 (Decided June 28, 2007).

[2] There are two main counterpoints to the pro-competitive view. Under uncertainty, conflicts can arise when a manufacturer must balance private incentives in supply with the need to provide insurance to retailers (Rey and Tirole, 1986). Second, Shaffer (1991a) considers oligopolistic retailers who use resale price maintenance (RPM) to dampen downstream competition in individual contracts with competitive manufacturers. See the excellent review of Mathewson and Winter (1998).

structure include supermarkets that carry both a national brand and a store brand (private label), convenience stores that sell gasoline and in-store consumption goods, and computer retailers that bundle essential components (such as processors and operating systems) with a set of commoditized components (such as DRAM, hard drives, and flat-panel displays).

Our analysis builds on several recent papers. Winter (1993) considers a single manufacturer that imposes vertical restraints on her duopoly retailers to elicit the optimal mix of prices and non-priced retail services. Absent contracts, retailers compete excessively in price and fail to provide a sufficient level of service; resale price maintenance (RPM), combined with a wholesale price elevated above marginal cost, simultaneously corrects both distortions. Rey and Vergé (2004) consider how RPM can be used by duopoly manufacturers to control pricing by duopoly retailers; RPM frees the wholesale price to be set at cost in order to avoid retail-manufacturer contract externalities that otherwise spur disadvantageous price competition.[3]

In the present setting, vertical restraints likewise serve to resolve retail market externalities; however, the essential difference is our focus on the joint pricing decisions of *multi-product* retailers who sell both a dominant manufacturer's good and a product produced by a competitive fringe. In this regard, our arguments are related to the substantial literature on the extension of monopoly power to other products through the use of tying arrangements (e.g., Whinston, 1990; Carbajo, et al., 1990; Shaffer, 1991b) or commodity bundling (e.g., Nalebuff, 2004; Mathewson and Winter, 1997; Degraba and Mohammed, 1999; MacAfee, et al., 1989). Innes and Hamilton (2006), for example, show how a dominant firm can use explicit cross-market controls on retailers' contracts to achieve an integrated multi-good monopoly outcome; the monopoly manufacturer dictates that suppliers of the rival good pay fixed "slotting fees" that elevate the good's wholesale price and thereby prompt retailers to charge the monopoly price. In practice, such explicit cross-market controls are likely to be infeasible, whether due to proscriptions of antitrust law or due to an inherent inability of the monopolist to observe and

[3] See also Marx and Shaffer (2004) and Rey, Thal and Vergé (2006), who study the structure of vertical contracts between a single manufacturer and differentiated duopoly retailers of the manufacturer's product.

verify retailers' cross-market conduct. Here, we focus on how a vertical restraint imposed on a manufacturer's *own product* can be used to extract rents from the market for another product, without stipulating any cross-market tying, bundling, pricing, or contract terms.

Vertical contracts that extract cross-market rents produce several notable symptoms. Irrespective of the relationship between the products in utility (complements, substitutes or independent goods), vertical restraints on the monopolist's product induce retailers to sign contracts with suppliers in the competitive fringe that involve fixed fees paid to the retailer ("slotting allowances"). Slotting allowances are prevalent in practice (FTC, 2003), and highly controversial. We show that their effects are often anti-competitive in a multi-product context.

In the case of goods that are weakly substitutable, independent, or weakly complementary in consumption, vertical restraints result in negative retail margins on the monopoly supplied good. Hence, our analysis offers a new explanation for loss-leader retail pricing that does not rely on coordination failures (Bagwell and Ramey, 1994), imperfect information (Lal and Matutes, 1994), heterogeneous consumers (Degraba, 2006), or product complementarities in multi-product monopoly (Bliss, 1988). When the goods are strong substitutes, the monopoly manufacturer sets her wholesale price below marginal cost, producing a type of "predatory accommodation" similar to that derived by Marx and Shaffer (1999), but for different reasons.

In Marx and Shaffer (1999), below-cost wholesale prices arise because a monopoly retailer can thereby extract rents from competing suppliers. Here, in contrast, distortions of the monopoly manufacturer's wholesale price are designed to counter the retailers' incentive to discount the rival good. With independent goods, the monopoly manufacturer sets a wholesale price above the (maintained) monopoly retail price in order to reduce the net retail profit per customer. This, in turn, reduces the retailer's incentive to attract customers from rivals by discounting the price of the competitively-supplied product. With substitute goods, reducing the wholesale price below marginal cost can be attractive, because a wholesale price reduction has two effects on the retailers' pricing incentives: it raises incentives to attract customers (favoring a lower retail price) and it raises incentives to shift consumption from the rival good to the high-margin monopoly good (favoring a higher retail price for the rival product). With strong substitutes, the second effect dominates and a below-cost wholesale price by the monopoly manufacturer prompts the retailers to raise the retail price of the fringe good.

We extend our observations to oligopoly settings where multiple manufacturers sign vertical contracts with multi-product retailers (see Section 6). Under oligopoly, each manufacturer ignores profit effects on rival manufacturers when pegging target retail prices. As a result, incentives for predatory accommodation are particularly harmful, leading to vertical restraints that raise equilibrium retail prices even higher than their monopoly counterparts. A below-cost wholesale price of one manufacturer – here designed to control the retail pricing of the fringe good – motivates the rival manufacturer to solicit higher retail prices for both its own product and the fringe good in order to shift consumption toward the high margin substitute.

2. The Model

Consider a "2x2x2" successive oligopoly structure with two goods, two manufacturers and two retailers. Good 1 is a "name brand" produced by a monopoly manufacturer and good 2 is a "generic brand" supplied to the retailers by a competitive fringe. Production of each good involves constant unit cost, denoted c^1 and c^2. For simplicity, retail costs are suppressed. The retailers engage in pure intermediation, purchasing goods at wholesale prices w^1 and w^2 in the upstream market and selling goods to consumers in the downstream market.

Due to economies of "one-stop" shopping that pervade retail environments (Bliss, 1988), each consumer purchases a consumption bundle (y^1, y^2) from a single retailer. Given her choice of retailer $j \in \{1,2\}$ and consumption bundle, a consumer obtains the utility,

$$(1) \qquad u(y^1, y^2) - \sum_{i=1,2} p_j^i y^i ,$$

where y^i is the quantity of good i purchased, and p_j^i is the price of good i at retail location j.

We assume $u(.)$ is increasing and concave with bounded first derivatives and that own product effects dominate cross-effects, $|\mathrm{dln}u_i / \mathrm{dln}y^i| > |\mathrm{dln}u_i / \mathrm{dln}y^j|$ for $j \neq i$, where $u_i \equiv \partial u(.)/\partial y^i$. The products can be substitutes, $u_{12} \equiv \partial^2 u(.)/\partial y^1 \partial y^2 < 0$, or complements $u_{12} > 0$, or independent goods, $u_{12} = 0$. The optimal consumption choice at retailer j yields the indirect utility,

(2) $$u_j^* \equiv u^*(p_j^1, p_j^2) = \max_{\{y^1, y^2\}} u(y^1, y^2) - \sum_{i=1,2} p_j^i y^i .$$

A consumer's choice between retailers is determined in a standard Hotelling framework, with the two retailers located at either end of the unit interval.[4] Consumers are uniformly distributed on this interval and incur preference (travel) cost of t per unit distance. The location $\theta \in [0,1]$ represents a consumer's distance from retailer 1, and $(1-\theta)$ her distance from retailer 2. This, a θ–type consumer obtains net utility $u_1^* - t\theta$ if purchasing from retailer 1 and $u_2^* - t(1-\theta)$ if purchasing from retailer 2. Given retail prices for each good at each retailer, a consumer of type

$$\theta^*(u_1^*, u_2^*) = (1/2) + [(u_1^* - u_2^*)/(2t)]$$

is indifferent between the retailers, and the market is partitioned into consumer types $\theta \leq \theta^*(u_1^*, u_2^*)$, who purchase both goods from retailer 1, and consumer types $\theta > \theta^*(u_1^*, u_2^*)$, who purchase both goods from retailer 2.

3. Collective Optimum and No Contract Outcomes

Absent contracts, the dominant firm sets a wholesale price w^1 and the competitive fringe prices at cost, $w^2=c^2$. Given these wholesale prices, duopoly retailers compete in retail prices. In this Section, we examine how the outcomes from this competition depart from the collective optimum for manufacturers and retailers. We then characterize the role of vertical restraints in aligning incentives between a monopoly firm and retailers of her product. Throughout, we assume that retailers cannot implement exclusive territories that would split the consumers between them, void any competition for customers, and thereby lead trivially to monopoly (collectively optimal) retail pricing.[5] For most examples in practice, exclusive territories are infeasible because consumers of a given type θ cannot be compelled to buy from a given retailer.

A vertically integrated monopolist solves:

---

[4] We suppress retailers' choices of location because these choices are long-run in nature, are therefore likely to precede the contractual decisions of interest in this paper, and, to a large extent, are based on considerations outside of our model, such as rents and the size and location of consumer markets.

[5] We therefore also assume that all consumers are served in this retail market, with t not so large as to foreclose the mid-point (θ=1/2) consumer.

(3) $$\max_{p^1,p^2} \sum_{i=1,2} (p^i - c^i)\, y^i(p^1, p^2) \equiv \Pi(p^1,p^2;c^1,c^2) \;\Rightarrow\; \{p^{1*}, p^{2*}\}$$

where $y^i(.) \equiv \text{argmax}\ \{u(y^1,y^2) - \sum_{i=1,2} p^i y^i\}$.[6] The solution to (3) yields the maximum profit available in this market, $\Pi^* \equiv \Pi(p^{1*},p^{2*};c^1,c^2)$, and associated demands, $y^{i*} \equiv y^i(p^{1*}, p^{2*})$.

Next consider the choice problem of retailer 1:[7]

(4) $$\max_{p_1^1,p_1^2} \pi_1(p_1^1,p_1^2;\bar{u}_2, w^1, w^2) \equiv \sum_{i=1,2}(p_1^i - w^i)y^i(p_1^1,p_1^2)\phi(p_1^1,p_1^2;\bar{u}_2)$$

$$= \phi(p_1^1,p_1^2;\bar{u}_2)\left[\Pi(p_1^1,p_1^2;c^1,c^2) - \sum_{i=1,2}(w^i - c^i)y^i(p_1^1,p_1^2)\right].$$

Π is defined in equation (3). Normalizing the number of consumers to one (without loss), $\phi(p_1^1,p_1^2;\bar{u}_2) = \theta^*(u^*(p_1^1,p_1^2),\bar{u}_2)$ is the market demand for retailer 1, given prices set by retailer 2 and the attendant utility level $\bar{u}_2$. The first-order necessary conditions for a solution to (4) are:

(5) $$\frac{\partial \pi_1}{\partial p_1^1} = \phi\left(\frac{\partial \Pi}{\partial p_1^1}\right) + \Pi\left(\frac{\partial \phi}{\partial p_1^1}\right) - \sum_{i=1,2}(w^i - c^i)\left[\phi\left(\frac{\partial y^i}{\partial p_1^1}\right) + y^i\left(\frac{\partial \phi}{\partial p_1^1}\right)\right] = 0$$

(6) $$\frac{\partial \pi_1}{\partial p_1^2} = \phi\left(\frac{\partial \Pi}{\partial p_1^2}\right) + \Pi\left(\frac{\partial \phi}{\partial p_1^2}\right) - \sum_{i=1,2}(w^i - c^i)\left[\phi\left(\frac{\partial y^i}{\partial p_1^2}\right) + y^i\left(\frac{\partial \phi}{\partial p_1^2}\right)\right] = 0\,.$$

where, using Roy's identity,

(7) $$\partial\phi/\partial p_1^i = (\partial u^*/\partial p_1^i)/2t = -\,y^i(p_1^1,p_1^2)/2t < 0.$$

Notice that the collective optimum (p^{1*}, p^{2*}) is achieved when the first term in each of equations (5) and (6) is equal to zero. Hence, the individual incentives of a retailer are compatible with the collective interest only when the remaining terms sum to zero. These terms correspond to two distortions. First, on the *inter*-retailer margin, higher prices by retailer 1 prompt consumers to switch to the rival retailer (the business stealing effect). This loss of store traffic is costly to the retailer, but of no concern to the vertically integrated chain. The business-stealing effect provides the retailer with an incentive to set each retail price below the level which maximizes joint profits. Second is an effect on the *intra*-retailer margin. To the extent that the retailer pays above-cost wholesale prices to its suppliers ($w^i > c^i$), retail price effects on

---

[6] $\Pi()$ is assumed to be concave for a relevant range of (p^1,p^2), as will be true if u is concave (as assumed) and has third-order derivatives that are sufficiently small relative to second-order derivatives.
[7] Choices of retailer 2 are symmetric and thus omitted.

demand have a smaller impact on retailer profit than on the profit of the vertically integrated chain, which faces true cost c^i. This "double-marginalization" generally induces the retailer to set prices above the level which maximizes joint profits.

Now, following Winter (1993) (and recalling that $w^2=c^2$), suppose that the wholesale price of good 1 is selected so that the business-stealing and double-marginalization effects exactly offset *for the good 1 retail price*,

$$w^1 - c^1 = \frac{\Pi(.)(\partial\phi / \partial p_1^1)}{\left(\phi(\partial y^1 / \partial p_1^1) + y^1(\partial\phi / \partial p_1^1)\right)} > 0. \tag{8}$$

With (8), the last terms in (5) vanish. However, the retailers compete for customers by jointly selecting prices for *both* goods and, in general, equation (8) does not also elicit the collectively optimal price of the fringe good 2. Formally, with w^1 set as in (8), let us evaluate equation (6) when p^2 equals its integrated optimum, p^{2*}:

$$\left.\frac{\partial\pi_1(p^{1*}, p^{2*}; \overline{u}_2, w^1, c^2)}{\partial p_1^2}\right|_{\text{eq. (8)}} = \frac{\phi\Pi^*\left[(\partial\phi / \partial p_1^2)(\partial y^1 / \partial p_1^1) - (\partial\phi / \partial p_1^1)(\partial y^1 / \partial p_1^2)\right]}{\phi(\partial y^1 / \partial p_1^1) + y^1(\partial\phi / \partial p_1^1)} < 0. \tag{9}$$[8]

The inequality in (9) implies that the retailer discounts the price of good 2 below p^{2*} in order to attract customers. Hence, absent contracts, a monopoly manufacturer cannot set her wholesale price to induce her retailers to set collectively optimal retail prices for *both* goods, (p^{1*}, p^{2*}).

4. Monopoly-Retailer Contracts

A. Analysis.

We now characterize contracts between the monopoly manufacturer and her retailers that elicit collectively-optimal retail prices. This task would be trivial if contracts could stipulate the retail price for the fringe product ($p^2= p^{2*}$) and punish any defections from this price. But, in practice, such a cross-product restraint would run afoul of prevailing antitrust law in most industrial countries, for instance violating both the "tying doctrine" and proscriptions against

---

[8] With $\partial y^1/\partial p_1^1<0$, $\partial\phi/\partial p_1^i < 0$, $\Pi^*>0$, and $\phi>0$, the inequality holds by inspection for independent goods and substitutes ($\partial y^1/\partial p_1^2 \geq 0$). For complementary goods ($\partial y^1/\partial p_1^2 < 0$), the inequality follows from our assumption, $|\text{dln}u_i / \text{dln}y^i| > |\text{dln}u_i / \text{dln}y^j|$ for $j\neq i$.

price fixing in U.S. case law, whether or not a “rule of reason” is applied.[9] Another direct cross-market control is a contract that requires retailers to levy a “slotting fee” on the rival manufactured good. Such a contract, recently considered by Innes and Hamilton (2006), would be difficult to enforce due to the dominant firm’s inability to observe and verify retail contracts with fringe suppliers.[10] We rule out such direct cross-product restraints as being either overtly anti-competitive or unenforceable, and consider monopoly-retailer contracts that instead have only three terms: resale price maintenance (RPM) for the dominant manufacturer’s good (requiring $p^1 = p^{1R} = p^{1*}$), a wholesale price (w^1) and a fixed tariff (f^1) to redistribute rents.[11]

We assume that these contract terms are determined by bargaining (see, for example, Macleod and Malcomson, 1995). Because the issue of interest here is the contract form that achieves the collective optimum, we do not describe the precise structure of the bargaining game. Instead, we assume that the game has a unique subgame perfect bargaining equilibrium that splits collective gains from contract implementation according to a known rule (as in Rubinstein, 1982; Shaked, 1987; and others).

We also assume, for now, that the retailers are unable to sign contracts with good 2 manufacturers. For instance, each retailer may be vertically integrated with a good 2 manufacturer as in the case of a supermarket in-store bakery. If the monopoly manufacturer imposes a vertical restraint on its retail price of $p^{1R} = p^{1*}$, then her retailers no longer optimize

---

[9] Although cross-market price controls differ from tied product sales, they represent a cross-market tie that transparently fixes prices. Prevailing case law in the U.S. proscribes such conduct and this is likely why, in practice, firms may use less transparent means to exercise cross-market control. For example, in the Northern Pacific Railway decision (Northern Pac. Ry. V. United States, 356 U.S. 1, 1958), the Supreme Court affirmed that “among the practices which the courts have heretofore deemed unlawful in and of themselves are price fixing … and tying arrangements.” Under the rule of reason (as endorsed by Justice O’Connor, see Collin and Wright, 2006), a cross-market tying restriction is judged to run afoul of antitrust law if the restriction has anti-competitive effects in the tied market, as is the case with a cross-market restraint that supports an elevated market price for the rival good.

[10] It generally would be possible to enforce such implicit commitments in repeated games with complete information; however, such considerations are beyond the scope of this study.

[11] With these simple contracts, we will show that the monopolist can achieve the collective optimum. Hence, there is no loss in generality from restricting contracts to this form. Equivalently, the vertical restraint could involve a good 1 quantity provision (e.g., at the level $y^1 = y^1(p^{1*}, p^{2*})/2$ in the symmetric two retailer case) in place of RPM (see Reiffen, 1999). For more on the equivalence between various forms of vertical restraints in a deterministic setting, see Mathewson and Winter (1984). For simplicity, we assume that contractual wholesale prices are observable to both retailers. For complications that arise when contracts are unobservable, see Cremer and Riordan (1987) and O’Brien and Shaffer (1992).

over the good 1 price, and the integrated optimum can be attained if a wholesale price, w^1, can be found to induce the duopoly retailers to select $p^2 = p^{2*}$ per equation (6). By inspection (and with $w^2 = c^2$ by construction), the wholesale price that achieves this integrated optimum is

$$w^1 - c^1 = \frac{\Pi(.)(\partial\phi / \partial p_1^2)}{\left(\phi(\partial y^1 / \partial p_1^2) + y^1(\partial\phi / \partial p_1^2)\right)}. \tag{10}$$

In a symmetric retail market equilibrium (where $\phi = ½$), equation (10) reduces to

$$w^1 - c^1 = \Pi^* y^{2*} / \delta, \tag{11}$$

where $y^{i*} = y^i(p^{1*}, p^{2*})$, i=1,2, is the equilibrium quantity of brand i sold by each retailer in the collective optimum; $\Pi^* = \Pi(p^{1*}, p^{2*}; c^1, c^2)$ is integrated maximal profit (from (3)); and $\delta \equiv y^{1*}y^{2*} - t(\partial y^{1*}/\partial p^2)$, Notice that δ is positive in the case of independent and complementary goods ($\partial y^{1*}/\partial p^2 \leq 0$), but can be negative when the goods are highly substitutable.

Definition: Substitute goods 1 and 2 are *weak substitutes* when $\delta = y^{1*}y^{2*} - t(\partial y^{1*}/\partial p^2) > 0$ and *strong substitutes* when $\delta < 0$.

In the case of independent goods, $\partial y^1/\partial p^2 = 0$ and the wholesale price in (11) is selected so that $(w^1 - c^1)\, y^{1*} = \Pi^*$. This is an intuitive result. Because the wholesale price of the fringe product is set at marginal cost, $w^2 = c^2$, the retailers depart from the collective optimum due only to the business-stealing incentive. Each retailer wishes to discount the price of good 2 to attract customers, and this incentive is entirely eliminated when the dominant firm selects her wholesale price to fully extract variable per-customer profit from the competing retailers. To do so, sales of product 1 are made below invoice ($w^1 > p^{1*}$) – a "loss leader" outcome – and each retailer's loss on product 1 exactly offsets her gain on sales of the fringe good 2 at the integrated price p^{2*}.

When the retail goods are not independent, the wholesale price must also correct for the incentive of retailers to shift consumers between products on the intra-retailer margin. Here, the sign of δ is crucial, reflecting the tension between two effects on good 2 retail pricing. Specifically, consider raising w^1 above c^1. The reduction in per-customer rents dampens retailers' business-stealing incentives, and this favors a higher p^2. However, for substitutes, the narrower retail margin on good 1 also reduces the opportunity cost of siphoning off sales of good 1 by offering a price discount on good 2; this "siphoning effect" favors a lower p^2.

In the case of *strong substitutes*, the siphoning effect dominates and the contract must combine a vertical restraint with a *lower* w^1 (below c^1) to induce retailers to raise the price of product 2. Accordingly, with $w^2 = c^2$ and $w^1 < c^1$, per-customer retail profit is positive, $\Pi = \Pi(p^{1*}, p^{2*}; w^1, c^2) > 0$. In the case of *weak substitutes*, the business-stealing effect dominates and, hence, the monopolist raises w^1 above c^1 to counter under-pricing of good 2. With w^1 raised above c^1, a higher p^2 yields retailers less benefit in increased good 1 profit (on the intra-retailer margin) than it does for the integrated firm; hence, if retailers also have incentives to steal customers, they will under-price, setting $p^2 < p^{2*}$. Only if retailers instead have incentives to rid themselves of customers (with $\Pi<0$) can integrated pricing incentives be restored.[12]

Note that the sign and magnitude of δ depends upon both the strength of the cross-price effect on demand, $\partial y^1/\partial p^2$, and on retailer differentiation (as measured by the travel cost t). When t is high, implying little inter-retailer rivalry, substitutes are *strong* ($\delta<0$). Conversely, when t is low, substitutes are *weak* ($\delta>0$). Put differently, t acts as an implicit weight on the relative strength of the siphoning (vs. business stealing) effect. In a perfectly contestable retail market (t=0), the business-stealing effect is all that matters and, to induce monopoly pricing, must be completely eliminated with a zero profit per customer (Π=0). As t rises, business stealing incentives decline. This reduces the incentive to discount the retail price of good 2 and can thus reduce the wholesale price deviation from cost needed to correct the under-pricing.[13]

In the event that $\delta = 0$, equation (11) has no bounded solution. For this case, the business-stealing and siphoning effects exactly offset, so that changes in the wholesale price w^1

[12] If the goods are complements, the "siphoning effect" also favors a higher p^2. Hence, setting w^1 above c^1 unambiguously prompts the retailer to elevate p^2, thereby countering the under-pricing that otherwise results from business-stealing incentives. In the monopolist's optimum, w^1 is elevated just far enough to exactly offset the *positive* business-stealing incentives that result from *positive* per-customer profit ($\Pi>0$).

[13] Formally, $d(w^1-c^1)/dt\,|_{\text{eq. (11)}} \overset{s}{=} \partial y^1/\partial p^2$, which is negative for complements (where $w^1-c^1>0$) and positive for strong substitutes (where $w^1-c^1<0$). For weak substitutes, (w^1-c^1) rises with t. The reason is that, as t rises, the extent of under-pricing falls, but the effectiveness of an increased wholesale price (w^1), in spurring a higher p^2, also falls. The second effect dominates, requiring a higher wholesale price deviation in order to fully correct under-pricing.

do not alter the retailers' pricing decision for good 2. In this Section, we assume that δ is bounded away from zero. We relax this assumption in Section 5, where retailers contract with suppliers of the fringe good. In this setting, a dominant manufacturer can use RPM and two-part tariffs to acquire horizontal control over a competitive fringe for all values of δ (including $\delta = 0$).

In summary, we have the following outcomes (see Table 1):

Proposition 1. Suppose that each retailer is vertically integrated with a fringe manufacturer so that $w^2 = c^2$. Then the dominant firm can achieve the collective optimum ($p^1 = p^{1*}$, $p^2 = p^{2*}$) using a contract that imposes a vertical restraint ($p^{1R} = p^{1*}$) and sets: (*i*) $w^1 < c^1$ when the goods are strong substitutes ($\delta<0$), and (*ii*) $w^1 > c^1$ when the goods are weak substitutes, complements, or independent ($\delta>0$). With strong substitutes or complements, per-customer retail profit is positive in equilibrium ($\Pi>0$); with independent goods, it is zero ($\Pi=0$); and with weak substitutes, it is negative ($\Pi<0$). Negative, zero, and small positive per-customer retail profit (for weak substitutes, independent goods, and weak complements) is achieved with loss-leader pricing ($w^1 > p^1 = p^{1*}$).

(Table 1 here)

B. A Loss-Leader Example: Gasoline Stations and Convenience Stores.

Consider the case of an oil company that distributes gasoline through dealer-operated stations that contain convenience stores ("quick stops"). The stations procure in-store consumption commodities from a set of competitive industries, which we assume can be aggregated into a composite consumption good. The in-store composite good is weakly complementary, weakly substitutable or independent from gasoline in consumption, and, consistent with our model, economies of multi-product purchases arise that favor the joint purchase of gasoline and consumer goods.[14] Our model predicts RPM contracts for gasoline that exhibit "loss leadership" ($w^1 > p^1$).

In practice, gasoline at convenience stores is widely recognized to be a loss-leader,

[14] Per-customer gasoline demand is determined by considerations such as automobile size that are unlikely to be strongly related to a consumer's demand for convenience products.

especially in urban areas.[15] Indeed, a number of State statutes in the U.S. prohibit below-cost gasoline pricing, for instance Tennessee's Petroleum Trade Practices Act and Florida's Motor Fuel Marketing Practices Act.

There are three types of arrangements for marketing of branded gasoline: (*i*) company operated stations, (*ii*) lessee dealerships, and (*iii*) dealer-owned stations. The prevalence of these different arrangements varies by region. For example, on the U.S. West Coast, lessee-dealer sales represent almost 50 percent of the market (Meyer and Fischer, 2004). In some States in the U.S., company-operated dealerships are prohibited by "divorcement" statutes.

Our analysis is most relevant to lessee dealerships and arguably branded dealer-owned stations that also contract with the central marketing company. Lessee dealer contracts typically involve two types of fixed payments to the brand company, a fixed purchase price for the franchise and periodic lease payments. In addition, contracts stipulate the dealer tank wagon (DTW) wholesale price for gasoline purchases from the company, and involve minimum volume requirements (Meyer and Fischer, 2004). Minimum purchase requirements are also common in contracts with dealer owned stations (www.state.hi.us/lrb/rpts95/petro/pet4.html).

In these contracts, the minimum volume requirements play the same role as the RPM modeled in this paper when the resale price (p^1) is below the wholesale (DTW) price (w^1). In particular, they can compel the station to make gasoline a loss leader, and compensate with profitable convenience store sales, in order to temper incentives for business stealing from other stations of the branded marketer and perhaps also those of other colluding gasoline companies. The implied purpose of the practice is to curb incentives for the gasoline station dealers to discount convenience store products to the detriment of integrated monopoly profit.

If gasoline and convenience goods are independent in consumption, our model predicts either no fixed transfers or fixed payments from the branded monopolist to lessee dealers. There

[15] Jeremy Bulow makes this point in a recent interview (Eunson, 2002). See also Motley Fool (2004, http://www.fool.com/investing/small-cap/2004/09/16/gasoline-fuels-7elevens-growth.aspx), where the high retail margins on in-store convenience items are contrasted with the negative retail margins for unleaded gasoline. Note that urban gasoline stations have significant repeat custom and, hence (consistent with our model), customers with good information about store prices.

are two reasons why observed fixed payments may go in the other direction. First, if the goods are weak complements, our model predicts loss leadership and positive per-customer profit that, if the oil company has all of the bargaining power, would be rebated by dealers to the brand company. Second, with inter-brand competition (e.g., Exxon vs. Texaco), a given brand-name oligopolist will temper dealers' business-stealing incentives (to avoid intra-brand customer poaching), but not eliminate them (to encourage inter-brand customer poaching); in this case, even with independent goods, we would expect positive per-customer retail profit that would be rebated to the brand company, but loss-leading to reduce intra-brand business stealing.

C. Minimum vs. Maximum RPM.

Although antitrust policy toward resale price restraints is fluid and varies across jurisdictions, presumed anti-competitive effects that often determine the legality of these practices are judged by whether they raise prices or not (see Comanor and Rey, 1997). Under such a criterion, *minimum* resale prices are illegal (because they can only serve to raise prices), but *maximum* resale prices are not. In the U.S., for example, *minimum* RPM was per se illegal before June 2007, while *maximum* RPM has been judged by the less restrictive *rule of reason* since 1997.[16] In our model, equilibrium resale price restraints can take either form:

Corollary 1. The optimal contract of Proposition 1 requires *minimum* resale prices ($p^1 \geq p^{1*}$) in the case of strong substitutes and *maximum* resale prices ($p^1 \leq p^{1*}$) in all other cases.

For strong substitutes, the below-cost good 1 wholesale price would, absent restraint, spur retailers to charge a good 1 retail price that is below the monopoly level p^{1*}. Conversely, for weak substitutes, loss-leader wholesale pricing would lead retailers to charge a higher retail price, absent restraint. While minimum RPM is therefore required for strong substitutes, and maximum RPM is required for weak substitutes, judging the anti-competitive effects of the

[16] Since 1911, minimum RPM has been deemed per se illegal in the U.S. (*Dr. Miles Medical Co. v. John D. Park and Sons*, 220 U.S. 373, 1911); in 1968, maximum RPM was also judged to be a per se violation of the Sherman Act (*Albrecht v. Herald Co.*, 390 U.S. 145, 1968). In two more recent decision, however, the Supreme Court has ruled that both forms of RPM should be judged by a "rule of reason," in 1997 for maximum RPM (*State Oil v. Khan*, 522 U.S. 3, 1997), and on June 28, 2007 for minimum RPM (*Leegin Creative Leather Products, Inc. v. PSKS, Inc.,* Slip Op. No. 06–480).

vertical contract is more complicated than the "minimum vs. maximum" distinction suggests. Consider banning maximum RPM (for weak substitutes). Absent RPM, the dominant firm selects a wholesale price (w^1) above the level that elicits the monopoly price p^{1*} in order to induce her retailers to charge a higher retail price for the fringe good. The optimal two-part contract balances the marginal cost of the good 1 price distortion ($p^1 > p^{1*}$) on profits with the marginal benefit of stimulating an increase in the price of good 2 ($p^2 < p^{2*}$). Hence, relative to the optimal RPM contract that supports (p^{1*}, p^{2*}), the good 1 price rises and the good 2 price falls. Banning *maximum* RPM can raise or lower economic welfare depending upon which effect dominates.

For strong substitutes, in contrast, both effects work in the same direction. Now, with the siphoning effect dominating, the monopoly manufacturer sets her wholesale price (w^1) *below* the level that elicits the monopoly price p^{1*} in order to spur a higher good 2 price. Again, an optimal two-part contract balances the marginal cost of the good 1 price distortion ($p^1 < p^{1*}$) with the marginal benefit of raising the good 2 price ($p^2 < p^{2*}$), but this trade-off now leads to lower prices for both goods. Banning *minimum* RPM is thus pro-competitive. In sum:

Corollary 2. Banning *minimum* RPM lowers both retail prices ($p^1 < p^{1*}$, $p^2 < p^{2*}$) and thus raises economic welfare.[17] In contrast, banning *maximum* RPM can lead to a higher retail price for one good and a lower retail price for the other good.

5. Retailer-Fringe Contracts

A. Analysis.

We now turn to the possibility of retailer contracting with suppliers of the fringe good. Vertical separation from the competitive manufacturers of product 2 is generally desirable for a retailer because it permits contracts that soften downstream price competition (Shaffer, 1991a). There is a similar incentive for separation between retailers and fringe suppliers in a multi-product retail environment. However, there is also a caveat: Unlike monopoly-retail contracts,

[17] Additional (plausible) regularity conditions are needed for proof of this result. See Appendix.

the terms of which are designed to control retail pricing for mutual advantage, retail-fringe contracts essentially involve a retailer's attempt to regulate its own pricing behavior. With repeated contracts over time, the ability to do so may be limited; that is, a retailer may recognize the average cost of fringe supply (true marginal cost) even when the contracted wholesale price is above cost. This would be true, for example, if the retailer cannot commit to an exclusive supply arrangement, in which case no fringe supplier will pay the up-front "slotting fees" characterized below. Often, however, we expect relatively long-term exclusive supply contracts to be possible (perhaps, in part, due to fixed costs of establishing a vertical relationship). Consistent with the literature, we make this assumption here.

Consider the following four stage game. First, the monopoly manufacturer selects a contract with each retailer that stipulates a wholesale price (w^1), fixed fee (f^1), and RPM ($p^{1R}= p^{1*}$), as before. Second, each retailer signs an independent contract with a fringe supplier that stipulates a wholesale price (w^2) and a fixed tariff (f^2).[18] Third, given observable wholesale prices from the first two stages, retailers compete in fringe good retail prices p^2 (with good 1 prices determined by contract). Finally, production and exchange occur.[19]

The analytical challenge is to show that, given a vertical restraint ($p^{1R}= p^{1*}$), there is a wholesale price w^1 that prompts the duopoly retailers to sign two-part contracts with fringe suppliers which in turn yield the collective optimum (p^{1*}, p^{2*}). Vertical separation occurs in this setting whenever retailers choose contracts with $w^2 \neq c^2$.

When contracting with fringe suppliers, a retailer can require a lump-sum payment of f_i^2, let suppliers compete in wholesale prices (w_i^2) to acquire the contract, and select among

[18] The restriction to two-part retailer-fringe contracts comes at no cost in generality; as of the second stage, a retailer can mimic outcomes from any non-linear fringe supply contract using a two-part equivalent. As is well known, this equivalence breaks down when there is uncertainty and asymmetric information that are absent in our model (see, for example, Mathewson and Winter, 1984; Martimort, 1996; Kühn, 1997).

[19] In principle, different orders of play are possible, for example simultaneous contracting between (i) retailers and the dominant firm, and (ii) retailers and fringe suppliers. The qualitative results derived below are robust to such alternative games. Note, however, that different orders of play will generally lead to lower integrated profit (because the monopoly manufacturer takes fringe wholesale markups as given and excludes these margins from joint profit maximization). As a result, in an expanded game wherein retailers choose the order of contracting, the order that we assume (monopoly-retailer contracting first) is an equilibrium outcome.

suppliers with the lowest prices on offer. Hence, in equilibrium, the terms of the contract will satisfy the (competitive) zero-profit condition,

(12) $$(w_i^2 - c^2) y^2(p_i^1, p_i^2)\phi(.) = f_i^2 .$$

Note that (12) applies whether the retail contract stipulates a fixed payment from the supplier to the retailer, $f_i^2 > 0$, or vice versa; hence, we impose no restriction on the signs of f_i^2 or, therefore, the wholesale markup, $w_i^2 - c^2$.

Let the per-customer retail profit function be defined by

$$\Pi(p^1, p^2; w^1, w^2) = \sum_{i=1,2} (p^i - w^i) y^i(p^1, p^2) .$$

Given retailer 1's contracted wholesale price with the supplier of good 2 (w_1^2), and the contractually pre-determined wholesale and retail prices of good 1 (w^1 and $p^1 = p^{1*}$), the retailer's optimal price for good 2 (p_1^2) is determined as follows:

(13) $$\max_{p_1^2} \Pi(p^{1*}, p_1^2; w^1, w_1^2)\, \phi(p^{1*}, p_1^2; \bar{u} = \mathrm{u}^*(p^{1*}, p_2^2)) \Rightarrow p^2(w^1, w_1^2; p_2^2)$$

where p_2^2 is the rival (second) retailer's price selection. Proceeding similarly with retailer 2, and equating the reaction functions, gives the equilibrium retail prices,

(14) $$p_1^{2e} = p^{2e}(w^1, w_1^2; w_2^2) \quad , \quad p_2^{2e} = p^{2e}(w^1, w_2^2; w_1^2)$$

where w_2^2 is the good 2 wholesale price faced by retailer 2. In order for the equilibrium in (14) to be locally stable at the integrated optimum, the following regularity restriction must hold:

Assumption 1. At $p^2(.) = p^{2*}$, $\partial p^2(w^1, w_1^2; p_2^2 = p^{2*})/\partial p_2^2 < 1$.[20]

Turning to the contract stage, each retailer chooses the fringe wholesale price w_i^2 to maximize profit subject to the subsequent price responses in (14). Given that supplier profits are rebated to the retailer in the retailer-fringe contract (12), retailer 1's problem is

(15) $$\max_{w_1^2} \mathrm{J} = \Pi(p^{1*}, p^{2e}(w^1, w_1^2; w_2^2); w^1, c^2)\phi(p^{1*}, p^{2e}(w^1, w_1^2; w_2^2); \bar{u} = u^*(p^{1*}, p^{2e}(w^1, w_2^2; w_1^2)))$$

The symmetric contract equilibrium solves (15), with $w_1^2 = w_2^2 = w^2$.

---

[20]Sufficient conditions for Assumption 1 to hold are that $\partial^2\Pi(p^{1*}, p^{2*}; w^1(w^2), w^2)/\partial(p^2)^2 < 0$ and $p^{2*}y^2(p^{1*}, p^{2*}) + t\,(dln\, y^2(p^{1*}, p^{2*})/dln\, p^2) \geq 0$, where $w^1(w^2)$ solves equation (16) below.

Now consider the problem of the monopoly manufacturer. Her challenge is to select a wholesale price w^1 such that, with the resulting equilibrium w^2 from the retail contracts solving (15), retailers set good 2 retail prices to maximize integrated profit, $p^2 = p^{2*}$. To characterize this solution, we seek a wholesale price pair (w^1, w^2) that simultaneously satisfies two conditions in the symmetric retail equilibrium: (*i*) $w_1^2 = w^2$ solves (15) when $w_2^2 = w^2$, and (*ii*) the resulting $p_1^2 = p^{2*}$ in the pricing stage solves (13) when $p_2^2 = p^{2*}$. Assuming the requisite second order conditions hold, differentiating (13) with respect to p_1^2 and evaluating at $p_1^2 = p_2^2 = p^{2*}$ gives

$$\text{(16)} \quad F_1(w^1, w^2) = -\Pi(p^{1*}, p^{2*}; w^1, w^2)\, y^2(p^{1*}, p^{2*}) - t\sum_{i=1}^{2} (w^i - c^i)\, \partial y^i(p^{1*}, p^{2*})/\partial p^2 = 0.$$

When $\delta \neq 0$, equation (16) has the closed form solution $w^1(w^2)$ that yields the collectively optimal price selection for good 2, $p^2 = p^2(w^1, w^2; p_2^2 = p^{2*}) = p^{2*}$.

Similarly, to solve (15), we take the first order condition, use (13)-(14) to expand terms, and evaluate when $w_1^2 = w_2^2 = w^2$ and $p^{2e}() = p^{2*}$ (by (16)) and $\phi = 1/2$ (a symmetric equilibrium):[21]

$$\text{(17)} \quad F_2(w^1, w^2) = \Pi(p^{1*}, p^{2*}; w^1, w^2)\, y^{2*}(\partial p^2()/\partial p_2^2)$$
$$+ (w^2 - c^2)\,\{[t\, \partial y^{2*}/\partial p^2] - (y^{2*})^2(1 - (\partial p^2()/\partial p_2^2))\} = 0,$$

where $\partial p^2()/\partial p_2^2 = \partial p^2(w^1, w^2; p_2^2 = p^{2*})/\partial p_2^2$.

Inspection of conditions (16) and (17) results in the following:

Proposition 2. If the two goods are independent in consumption ($\partial y^{1*}/\partial p^2 = 0$), then the collective optimum is supported by $w^2 = c^2$ and $w^1 > p^{1*}$ such that $\Pi(p^{1*}, p^{2*}; w^1, c^2) = 0$.

For independent goods, our results replicate those derived above. The monopoly manufacturer writes a loss-leading vertical contract ($w^1 > p^{1*}$) with her retailers that eliminates the retailers' variable profit per customer ($\Pi = 0$). With zero variable profit, the retailers gain no advantage by signing contracts with fringe suppliers; a contract with a fringe supplier can be

---

[21] From (2) and the definition of ϕ in (4), $[\partial\phi()/\partial\bar{u}][\partial u^*(p^{1*}, p^2)/\partial p^2] = y^2()/2t$. With $\Pi(.; w^1, c^2) = \Pi(.; w^1, w^2) + (w^2 - c^2)\, y^2(p^{1*}, p^2)$, we have from (13), $(d/dp^2)[\Pi(.; w^1, c^2)\phi()] = (w^2 - c^2)\,\{\partial y^2()/\partial p^2 \phi + y^2()(\partial\phi/\partial p^2)\}$, where $\partial\phi/\partial p^2 = -y^2()/2t$. Differentiating (14), $\partial p^{2e}(w^1, w_2^2; w^2)/\partial w^2 = [\partial p^{2e}(w^1, w^2; w_2^2)/\partial w^2](\partial p^2(w^1, w^2; p_2^2)/\partial p_2^2)$ when $w^2 = w_2^2$. Substituting into the first order condition for (15) when $w^2 = w_2^2$ and $p^{2e}() = p^{2*}$ gives (17).

used to steal business from the rival retailer, but shifting customers no longer shifts rent. Retailer-fringe contracts do not arise, and Proposition 1 applies.

The above logic does not extend to goods that are not independent in consumption ($\partial y^1()/\partial p^2 \neq 0$). For substitutes (complements), retail losses on good 1 will prompt retailers to lower (raise) the good 2 price below (above) its optimal level in order to shift consumption away from good 1 (on the intra-retailer margin). For these cases, we have the following:

Proposition 3. When $\partial y^{1*}/\partial p^2 \neq 0$, there is a $w^{2*}>c^2$ such that $(w^{1*}, w^{2*})=(w^1(w^{2*}), w^{2*})$ solve equations (16) and (17). Hence, the collective optimum can be achieved by vertical restraints on the retailers of the dominant manufacturer's own good. The optimal contract prompts the retailers to set positive tariffs ($f^{2*}>0$) on fringe manufacturers.

Under the monopolist's optimal contract, we demonstrate below that a retailer's profit per-customer (Π) can be either positive or negative in equilibrium (much as in Proposition 1). The novel aspect of Proposition 3 is that, *in either case*, retailers sign contracts with fringe suppliers that stipulate above-cost wholesale prices for good 2, $w^2> c^2$. When per-customer profit is positive in the retail market ($\Pi>0$), the elevated wholesale price implicitly commits the retailer to charge a higher good 2 retail price, which is advantageous to the retailer because her rival responds with a higher good 2 price (Shaffer's (1991a) insight). An elevated wholesale price is also advantageous when per-customer profit is negative in the retail market ($\Pi<0$). This is because the rival now responds with a *lower* retail price, which rids the contracting retailer of costly customers on the inter-retailer margin.[22] By (12), the above-cost wholesale price is supported by positive tariffs on fringe suppliers ($f^2>0$).

Understanding these retailer-fringe contractual incentives, the monopolist selects her wholesale price (w^1) so as to elicit integrated good 2 pricing. Specifically, we define the

[22] The intuition for this rival retailer response is rather subtle. Speaking somewhat loosely, when $\Pi<0$, retailers set p^2 at a high level that trades off two effects on retail profit: (i) the marginal benefit of a higher p^2 in ridding the retailer of costly customers, and (ii) the marginal cost of higher p^2 in lowering per-customer profit Π. Now, when the opposing retailer raises its price (due to an elevated wholesale price), the second effect – the marginal cost of a higher p^2 – becomes larger because the retailer's market share rises. Because the first (customer shedding) benefit of higher p^2 is unchanged, the retailer responds with a lower price.

retailer's profit per-customer under the optimal contract as $\Pi^{**} = \Pi(p^{1*}, p^{2*}; w^{1*}, w^{2*})$, and make use of the parameter δ defined in (11),[23]

(18) $\delta = y^{1*}y^{2*} - t\, \partial y^{1*}/\partial p^2 \overset{s}{=} - dw^1(w^2)/dw^2 \overset{s}{=} d\Pi(p^{1*}, p^{2*}; w^1(w^2), w^2)/dw^2$.

Recall that $\delta > 0$ in the case of complementary goods, but that, for substitute goods, $\delta > 0$ when the goods are *weak substitutes* and $\delta < 0$ when the goods are *strong substitutes*.

Proposition 4. (*i*) When the retail goods are complements ($\partial y^{1*}/\partial p^2 <0$), $\Pi^{**} >0$ and $w^{1*}>c^1$; (*ii*) when the retail goods are weak substitutes ($\partial y^{1*}/\partial p^2 >0$), $\Pi^{**} <0$ and $w^{1*}> p^{1*}>c^1$; and (*iii*) when the retail goods are strong substitutes ($\partial y^{1*}/\partial p^2 >0$), $\Pi^{**} >0$ and $w^{1*}<c^1$.

The intuition for Proposition 4 is straightforward. Consider the starting point of marginal cost good 1 pricing, $w^1=c^1$. When retailers raise their good 2 wholesale prices with fringe contracts ($w^2>c^2$), they reduce – but do not eliminate – their incentive to discount good 2. There are two reasons: (*i*) on the inter-retailer margin, customers are less profitable, which decreases incentives to discount p^2 in order to attract customers; and (*ii*) on the intra-retailer margin, the opportunity cost of reducing good 2 sales, by elevating p^2, falls. As a result, the extent to which the good 1 wholesale price needs to depart from cost in order to correct retailers' under-pricing of good 2 is also reduced, but not eliminated. The directions of distortion identified in Proposition 1 are the same (and for the same reasons), but the extent of distortion declines.

Corollary 3. Retailer-fringe contracting reduces the extent to which good 1 wholesale prices depart from cost, $| w^1 - c^1|$, implying a higher wholesale price when the goods are strong substitutes, and a lower wholesale price when the goods are weak substitutes or complements.

The preceding results are derived under the premise of a non-zero δ. However, unlike the case of vertically integrated retailers considered in Section 4, the $\delta\neq0$ restriction is not necessary for the exercise of horizontal control when the retailers are vertically separated from their good 2

---

[23] The first sign equality in (18) follows from differentiation of (16). The second follows from substitution for $(dw^1(w^2)/dw^2)$ in the equation, $d\Pi\,()/dw^2 = - y^1\,(dw^1(w^2)/dw^2) - y^2$, and appeal to our initial assumption that $|d\ln u_1/d\ln y^1| > |d\ln u_1/d\ln y^2|$.

suppliers. Even when δ=0, there is an above-cost good 2 wholesale price, $\hat{w}^2 > c^2$, that elicits optimal good 2 retail pricing, $p^2 = p^{2*}$. Formally, evaluating (16) when $\delta = 0$ yields

(16')
$$\hat{w}^2 - c^2 = y^{2*}\Pi^*/\gamma > 0,$$

where $\gamma \equiv (y^{2*})^2 - t(\partial y^{2*}/\partial p^2) > 0$ is the good 2 counterpart to δ. This above-cost wholesale price can be supported by endogenous retailer-selected slotting fees that, in turn, are influenced by the dominant manufacturer's choice of the good 1 wholesale price. This chain of effect now becomes the channel for horizontal control. Intuitively, when w^1 is undistorted ($w^1 = c^1$), retailers set positive slotting fees for the usual reasons (Shaffer, 1991a). Due to business-stealing incentives, however, these fees do not fully resolve interbrand competition, and the resulting wholesale prices are set below the level necessary to induce monopoly pricing of good 2 ($w^2 < \hat{w}^2$). The monopoly manufacturer can increase retailers' incentives to raise slotting fees by increasing their per-customer profit; with a higher per-customer profit, a retailer reaps a greater strategic benefit from a higher wholesale price (w^2) because the resulting increase in the rival retailer's retail price (p^2) is more profitable. The desired increase in per-customer profit – and attendant incentive to elevate w^2 to $\hat{w}^2$ – can be achieved by lowering the good 1 wholesale price below cost ($w^1 < c^1$).

Proposition 5. When δ=0, the collective optimum can be achieved by a vertical restraint on the dominant manufacturer's good ($p^{1R} = p^{1*}$), and a below-cost wholesale price, $w^1 \in (-\infty, c^1)$, that supports the good 2 wholesale price, $\hat{w}^2 > c^2$, defined in equation (16').

For cases in which δ is close or equal to zero, Proposition 5 provides a new explanation for vertical separation between retailers and competitive suppliers.

B. Application to Supermarkets.

Supermarkets, drug chains and mass merchandisers frequently offer private label ("store brand") products that closely substitute for national brands.[24] Private label products in the

[24] In general, the term "private label" refers to any product in which a manufacturer enters into a relationship with a buyer to use the buyer's name on its product. Under this definition, private labels are sold in a wide variety of product categories including wine, credit cards, medical equipment, electronics, software, and website content (both

supermarket industry generate a significant share of total retail sales (22 percent in Europe and 16 percent in North America in 2002) and, in the U.S., have a greater market share than the leading manufactured brand in roughly 30 percent of all product categories.

Private labels are supplied to retailers in three different ways: (*i*) direct production by the retailer (e.g., an in-store bakery); (*ii*) by the manufacturer of a national brand (e.g., Coca Cola produces ASDA Cola in the U.K.); and (*iii*) by contract manufacturers specializing in private label production (e.g., Ralcorp produces various ready-to-eat cereals and crackers). In the first case, horizontal control can be achieved directly by vertical restraints without third party contracts (as in Proposition 1). In the second case, a single manufacturer can maximize collective rents over the national brand and private label, without the use of a vertical restraint, by jointly selecting wholesale prices. However, the third case is the most common. Indeed, the private label market is dominated by small, independent suppliers (Supermarket News, 1995).

Private label procurement at supermarkets typically occurs through the use of an in-house broker (IHB). IHBs assist supermarkets with their private label programs by selecting among potential private label suppliers and by providing services such as procurement, category management, quality control, label design, retail pricing and merchandising. Nearly 80 percent of private label purchases by U.S. supermarkets are brokered through IHBs at a cost ranging from 1 percent to 6 percent of sales (PLBuyer, 2004).

Consider a national brand manufacturer who imposes a vertical restraint to control the retail pricing of a supermarket private label. For strong substitutes, our model predicts the emergence of retailer contracts for private labels that involve lump-sum payments to supermarkets in exchange for elevated wholesale prices ($w^2 > c^2$). And indeed, evidence suggests that IHBs rebate a significant share of their brokerage commission to supermarket retailers. This is done either through direct cash payment from IHBs to their retailers or through "in kind" rebates, for instance by placing supermarket employees on their payrolls and acquiring

graphics and text). Here we choose to use the terms "private label" and "store brand" synonymously and focus on the case of supermarket private labels that are close substitutes for a national brand.

retail service functions previously performed by supermarket personnel. Marion (1998) estimates that up to 80-95 percent of the brokerage commission collected from private label suppliers by IHBs is rebated through lump-sum transfers to retailer accounts. To the extent that brokerage commissions on private label sales pass through to wholesale prices, this practice raises the wholesale price of private label products, $w^2 > c^2$.

Regarding contracts between national brand manufacturers and supermarket retail chains, our model predicts *minimum* resale price contracts that are per se illegal in Europe and, until 2007, in the U.S. (see Comanor and Rey, 1997). If RPM were allowed, we would expect to see below-cost wholesale pricing of the national brand ($w^1 < c^1$), and a tariff paid by retailers to the manufacturer of the national brand. However, we expect the proscription of RPM to instead spur legal two part contracts such that the national brand wholesale price (w^1) is above cost, but lower than the level that would otherwise prompt monopoly pricing (p^{1*}); the lower wholesale price is advantageous to the brand company because it yields a higher retail price for the substitute fringe good. Due to profit costs of lowering the monopoly brand wholesale price, the contract will never go so far as to elicit a monopoly price for the fringe good. A ban on minimum RPM thus yields strictly lower retail prices for both the branded and fringe products.

Given illegality of RPM, our model also indicates that supermarket-fringe "slotting allowance" / IHB-rebate contracts are anti-competitive. The direct effect of a retailer-fringe contract is to raise the retail price of good 2; however, there is also an indirect effect on good 1. The retailer-fringe contract reduces the incentive of manufacturer 1 to lower w^1 to achieve a higher p^2, and this raises the retail price of good 1 (see Corollary 3). Slotting allowances on suppliers in a competitive fringe can thus lead to higher retail prices for *both* goods.

6. Extensions

A. Oligopoly

We have assumed that good 1 is produced by a monopolist. Suppose instead that the upstream market for good 1 is populated by differentiated duopolists. We then have a three-good model that is otherwise the same as before. The three goods are the two duopoly-supplied products (close substitutes that we will denote by 1A and 1B) and the fringe good (2). We assume that the first two goods enter consumer utility $u(y^{1A},y^{1B},y^{2})$ in a symmetric way.

The duopoly competition is similar to that studied by Rey and Verge (2004), with the key difference here – the point of our paper – that there is a fringe market which the dominant firms seek to control. The key insight of Rey and Verge (2004) is that resale price maintenance can facilitate horizontal control of the retail market for the duopoly-supplied goods, relative to simple two-part contracts. Why? Above-cost wholesale prices that emerge under two-part contracts have two effects. First is the beneficial effect for the duopolists: they prompt the competing retailers to raise their prices closer to the monopoly level. Second, when simultaneously contracting with retailers, the duopolists take the rival's above-cost wholesale price as given; hence, they seek to maximize their own joint profit with the retailers, excluding the rival manufacturer's margin and thus leading them to support a retail price that is discounted from its monopoly level (ignoring the cost to the rival). When using RPM to control retail pricing rather than the wholesale price, manufacturers can set wholesale prices equal to marginal cost, which voids incentives to discount retail prices. Here, however, RPM does not void the duopolists' interest in distorting the wholesale price, which they use to control retail incentives for fringe good pricing. As a result, in equilibrium, retail prices are driven away from monopoly levels.

Consider the case in which retailers are integrated with a fringe supplier (as in Section 4). The duopolists simultaneously negotiate observable three-part contracts (maintained retail price, wholesale price and fixed transfer) with the two retailers. Denoting the associated retail and wholesale price pairs by (p^{1A},w^{1A}) and (p^{1B},w^{1B}), and assuming common manufacturer unit costs c for all three goods (for symmetry and to avoid clutter), retailers solve:

$$(19) \qquad \max_{p^2}\ [\,\Pi^*(p^{1A},p^{1B},p^2) - \sum_{i=1A,1B}(w^i-c)y^i(p^{1A},p^{1B},p^2)\,]\,\phi(p^{1A},p^{1B},p^2;\bar{u})$$

where $\Pi^*() = \sum_{i=1A,1B,2}(p^i-c)y^i(p^{1A},p^{1B},p^2)$ is fully integrated profit, $\bar{u}$ is consumer utility at the rival retailer, and $\phi = (1/2) + [(u^*(p^{1A},p^{1B},p^2)-\bar{u})/(2t)]$ is the retailer's market share. Given the rival contract, (p^{1B},w^{1B}), manufacturer 1A can use her wholesale price (w^{1A}) to control the retailers' choice of p^2 (per equation (19)).

We envision two simultaneous bargaining games, in each of which the two retailers negotiate with one of the duopoly manufacturers, taking the contracts with the rival manufacturer as given.[25] As before, we abstract from particulars of the bargaining games that determine how collective gains are split between the contracting parties, and assume instead that the parties maximize joint rents and share their gains using fixed transfers. Contracts between manufacturer 1A and her retailers thus solve the joint profit maximization problem,

$$\text{(20)} \qquad \max_{p^{1A},p^2} \Pi^{**}(p^{1A},p^{1B},p^2;w^{1B}) \equiv \Pi^*(p^{1A},p^{1B},p^2) - (w^{1B}-c)y^{1B}(p^{1A},p^{1B},p^2) ,$$

maximizing integrated profit less manufacturer 1B's margin.

Under plausible regularity restrictions, the duopoly retail price that solves problem (20) (p^{1A}) declines with the wholesale price w^{1B}.[26] A higher good 1A retail price, by stimulating demand for the substitute good 1B, yields profit gains to manufacturer 1B that rise with the wholesale price w^{1B}. Because these gains are ignored by manufacturer 1A and her retail contracting partners, the contracting manufacturer counters a rise in w^{1B} with a decrease in p^{1A}.

In a symmetric contract (and pricing) equilibrium, we have the retail prices from (20),[27]

$$\text{(21)} \qquad p^{1A} = p^{1B} = p^{1**}(w) \ , \quad p^2 = p^{2**}(w),$$

and the wholesale price $w = w^{1A} = w^{1B}$ that yields the latter fringe retail price from (19); that is, w solves the first-order condition for problem (19),

$$\text{(22)} \qquad -\Pi^{**}()y^2 + (w-c)\{y^{1A}y^2 - t(\partial y^{1A}/\partial p^2)\} = 0,$$

where all functions are evaluated at equilibrium prices. Equation (22) has two implications.

---

[25] We thereby sidestep potentially complex issues on the use of one duopolist's contract to extract rents from the other duopolist's contracting game.

[26] Sufficient conditions for this property are (*i*) consistent with second order conditions, $|\Pi^{**}_{2,2}| > |\Pi^{**}_{1A,2}|$, where $\Pi^{**}_{i,j} = \partial^2\Pi^{**}/\partial p^i \partial p^j$, and (*ii*) 1A and 1B are "stronger" substitutes than goods 1B and 2 in the sense that $\partial y^{1B}/\partial p^{1A} \geq |\partial y^{1B}/\partial p^2|$. Then (using second order conditions): $\partial p^{1A}/\partial w^{1B} |_{\text{eq. (20)}} \overset{s}{=} (\partial y^{1B}/\partial p^{1A}) \Pi^{**}_{2,2} - (\partial y^{1B}/\partial p^2) \Pi^{**}_{1A,2} < 0$.

[27] Problem (20) yields solutions, $(p^{1A+}(w^{1B},p^{1B}), p^{2+}(w^{1B}, p^{1B}))$. By symmetry, we can define the equilibrium price functions for common $w = w^{1A} = w^{1B}$: $p^{1**}(w) = p^{1A}$: $p^{1A} = p^{1A+}(w,p^{1A})$, $p^{2**}(w) = p^{2+}(w, p^{1**}(w))$. For stability of the retail pricing equilibrium, we assume that $\partial p^{1A+}/\partial p^{1B} < 1$ (c.f., Assumption 1).

First is the analog to equation (11):[28]

$$(23) \qquad w\text{-}c = \Pi^{**}()y^2/\delta^A \overset{s}{=} \delta^A,$$

where $\delta^A \equiv y^{1A}y^2 - t(\partial y^{1A}/\partial p^2)$. Second is the equilibrium retail profit per customer:

$$(24) \qquad \Pi^{***} \equiv \Pi^{**} - (w\text{-}c)y^{1A} = -\,t\,(w\text{-}c)(\partial y^{1A}/\partial p^2)/y^2.$$

Together, (23) and (24) directly imply the key qualitative results of Proposition 1.

The important difference here is that the equilibrium retail prices do not maximize collective profit, the unfortunate consequence of the duopolists' lack of coordination in their contracting with retailers. Ordinarily, the coordination failure of oligopolists has favorable implications for consumer prices relative to the monopoly outcome. This is not necessarily the case here. Consider, for example, when all three goods are strong symmetric substitutes. Then we have a symmetric price equilibrium, $p^{1A} = p^{1B} = p^2 = p^{1**}(w)$, with $dp^{1**}(w)/dw < 0$.[29] From problem (20), retail prices fall with the rival duopolist's wholesale price w because the ignored benefits of higher retail prices, in shifting custom to the rival duopoly good and thus raising the rival's profits, are higher when w is higher. Hence, with $w<c$ (by equation (23) with strong substitutes, $\delta<0$), all retail prices are higher than would be charged by a fully integrated monopolist. Manufacturers charge retailers a below-cost wholesale price in order to spur them to raise their price on the fringe good. The low wholesale price of the rival duopoly product inflates the retail margin for that good. The inflated margin in turn motivates the retailers to shift custom to the rival duopoly good by raising retail prices of the other substitute products above their monopoly levels. As a result, oligopolistic competition leads to an equilibrium outcome that is socially inferior to that which would emerge under monopoly.

Of course, this is not a general conclusion. With weak substitutes, for example, the wholesale price w is *raised* above cost in order to spur a higher good 2 retail price (problem

---

[28] The sign equality in (23) follows from $\Pi^{**}()>0$ in a symmetric equilibrium in order for the duopoly manufacturers to earn non-negative profit. ($\Pi^{**}\leq 0$ implies the contradiction of $w>c$ and zero manufacturer profit.)

[29] Here, by symmetry and second order conditions, we have , $0< -\Pi^{**}_{2,2} = -\Pi^{**}_{1A,1A} > |\Pi^{**}_{1A,2}|$ and $\partial y^{1B}/\partial p^{1A} = \partial y^{1B}/\partial p^2 > 0$, implying that $\partial p^{1A+}/\partial w^{1B}|_{(20)} <0$ (see note 26). Hence, with $\partial p^{1A+}/\partial p^{1B} < 1$ (note 27), $dp^{1**}(w)/dw = [\partial p^{1A+}/\partial w]/[1-(\partial p^{1A+}/\partial p^{1B})]<0$.

(19)). The elevated wholesale price in turn lowers contracting parties' optimal good 1 retail price (with $dp^{1**}(w)/dw < 0$), and, under plausible conditions, also lowers their optimal good 2 retail price ($dp^{2**}(w)/dw < 0$).

Proposition 6. Duopoly production of good 1 leads to: (i) below-cost wholesale prices ($w<c$) when the fringe good is a strong substitute ($\delta^A<0$), (ii) above-cost wholesale prices ($w>c$) otherwise, (iii) loss-leader pricing ($w> p^{1**}$) for independent goods, weak substitutes and weak complements, and (iv) retail prices that depart from the collective integrated industry optimum ($p^{1*}= p^{1**}(c)$, $p^{2*}= p^{2**}(c)$). With independent goods ($\partial y^{1A}/\partial p^2=0$), $p^{1**}< p^{1*}$ and $p^{2**}= p^{2*}$; with strong substitutes, all retail prices can be higher than monopoly counterparts ($p^{1**}> p^{1*}$, $p^{2**}> p^{2*}$); with weak substitutes ($\partial y^{1A}/\partial p^2>0$), all retail prices can be lower ($p^{1**}< p^{1*}$, $p^{2**}< p^{2*}$); and with complements ($\partial y^{1A}/\partial p^2<0$), the good 1 retail prices can be lower than monopoly levels, and the good 2 retail price higher ($p^{1**}< p^{1*}$, $p^{2**}> p^{2*}$).

As before, below-cost wholesale prices would spur "low" retail prices absent vertical restraints; hence, in the case of strong substitutes, a *minimum* RPM restraint is required to implement the duopoly equilibrium of Proposition 6. In the remaining cases (weak substitutes, independent goods, and complements), a *maximum* RPM restraint is required.

B. Retail Formats without Product Bundling.

Until now we have considered a spatial downstream market that involves multi-product transactions between consumers and retailers. This bundling of consumer purchases in a single retail transaction is a natural property in many, but not all, retail markets.

Is this bundling important for our results on horizontal control? For the case of independent goods (where $\partial y^1(.)/\partial p^2 = \partial^2 u(.)/\partial y^1 \partial y^2 = 0$), the answer is "yes." However, for all other cases, the answer is "no." Consider a generalized statement of our model, with product $k \in \{1,2\}$ demands at retailer $i \neq j$, $(i,j) \in \{1,2\}$, denoted by $D^k(p_i^1, p_i^2, p_j^1, p_j^2)$. Then three-part monopoly-retailer contracts (with maintained price p^{1R}, wholesale price w^1, and fixed transfer) yield the following retailer i choice for the fringe good price p_i^2, assuming the retailer has integrated good 2 production (for simplicity):

(25) $$\max_{p_i^2} (p^{1R}\text{-}w^1)\, D^1(p^{1R},p_i^2, p^{1R},p_j^2) + (p_i^2 - c^2)\, D^2(p^{1R},p_i^2, p^{1R},p_j^2).$$

Compare this choice to that for the fully integrated industry:

(26) $$\max_{p^1,p^2} \sum_{k=1,2} (p^k\text{-}c^k) D^k(p^1,p^2; p^1,p^2) \rightarrow (p^{1*}, p^{2*}).$$

Examining the respective first-order conditions, the solution to problem (26) is supported by contracts that set $p^{1R} = p^{1*}$ and w^1 that satisfy:

(27) $$- (w^1\text{-}c^1)(\partial D^1(.) /\partial p_i^2) = \sum_{k=1,2} (p^k\text{-}c^k)\, (\partial D^k(.)/\partial p_j^2).$$

Now suppose that consumers do not have multi-product purchase economies and therefore can buy one good from one retailer and the other good from the other retailer, without cost. If the two goods are also independent in consumption, the retailers' good 1 demands are completely invariant to good 2 prices, $\partial D^1(.) /\partial p_i^2 = \partial D^1(.)/\partial p_j^2 = 0$. Because good 2 demands depend on good 2 prices, $\partial D^2(.)/\partial p_j^2 > 0$, there is no wholesale price (w^1) that can satisfy condition (27) for this case. When the two goods are completely independent in *both* consumption and the shopping process, retail pricing of the monopoly good 1 and the fringe good 2 are also independent; hence, the monopolist cannot control the latter with her contract on the former. In this case, there is a motive for cross-market control, but these controls must be explicit, for instance through the use of tying arrangements, commodity bundling, or cross-market slotting fees of the form described by Innes and Hamilton (2006).

If the two goods are *not* independent in consumption, then $\partial D^1(.)/\partial p_i^2 \neq 0$, and equation (27) has a solution. Our main conclusions then stand. Hence, any jointness between products in either consumption or shopping enables the monopolist to exert horizontal control using a three part vertical contract with no explicit cross-market terms. Moreover, with $\partial D^k(.)/\partial p_j^2 > 0$, the wholesale cost distortion is the same as described in Proposition 1, $(w^1\text{-}c^1) \overset{s}{=} \delta \equiv - \partial D^1(.) /\partial p_i^2$.

7. Conclusion

In this paper, we study how a vertical restraint by a manufacturer of one good can be used to simultaneously control the retail pricing of another good, resulting in the extension of monopoly power to a second market. The central elements required for this to occur are: (*i*) a multi-product retail market in which oligopoly retailers compete in common goods; (*ii*) a monopoly (vs. oligopoly) manufacturing industry in the upstream market for one of the goods; and (*iii*) an element of jointness between the goods in consumer demand. Jointness can arise either in consumption (so that a consumer's demand for one good is affected by price of the other) or in "shopping" (so that economies exist in buying both products from the same retailer). Because such jointness exists in a wide range of economic settings, our analysis suggests that antitrust scrutiny is warranted for vertical restraints, even when direct mechanisms for cross-market control, for instance explicit tying and price fixing arrangements, are not employed.

We identify several symptoms of vertical contracts used to exert horizontal control, including predatory (below cost) wholesale pricing for strong substitutes, retailer-driven "slotting fees" for competitive suppliers of the rival good, and loss-leader retail pricing for weak substitutes, weak complements, and independent goods. Some of these practices are commonplace; for example, transfers from contract manufacturers to retailers in the form of discounted loans, technology, and demonstration equipment are common in many retail settings. For the case of supermarket retailing, moreover, the available evidence indicates that direct cash transfers occur through rebates paid to retailers by in-house-brokers of their private labels.[30]

All of these symptoms, and the vertical restraints that underpin them, are the subject of the ongoing antitrust policy debate. For example, statutes prohibit below-cost retail pricing in a number of European countries and for gasoline in a number of U.S. States (Allain and Chambolle, 2005). Antitrust law also proscribes predatory pricing. As elucidated in the case of *Brooke Group Ltd. V. Brown & Williamson Tobacco Corp.* (92-466, 509 U.S. 209, 1993), a prerequisite for below-cost pricing to be deemed predatory and thus illegal under U.S. law is that the pricing firm has a reasonable prospect of recouping losses. This condition is satisfied in the vertical contracts characterized in this paper (even though it was not satisfied in the *Brooke*

[30] The related practice of charging slotting allowances to suppliers has drawn recent regulatory attention in the U.S. (FTC 2003), although no explicit linkage has been made to the use of vertical restraints.

case), because losses are recouped with fixed transfers between the parties.

Are legal proscriptions of below-cost pricing beneficial to society? In our model, we distinguish between below-cost wholesale pricing (the "predatory accommodation" associated with strong substitutes) and below-cost retail pricing (the loss-leading of weakly related goods). A proscription against wholesale predation prevents the monopolist from lowering her wholesale price below cost in order to spur a higher retail price for the rival good. Hence, if RPM is allowed, the wholesale price will be set equal to cost, and the rival retail price will be set below its monopoly level. If the two goods' retail prices are strategic substitutes (in the sense that a higher price for one good spurs retailers to charge a lower price for the other good), the (maintained) retail price on the monopoly good will also be set below the monopoly level to induce a higher retail price on the fringe good. Similarly, a proscription against retail predation (loss leading) prevents the monopolist from raising her wholesale price above the (maintained) retail price. When the proscription binds, the wholesale price is set equal to retail price and the retail price of the fringe good is again set below the monopoly level. If the retail prices are strategic substitutes, for instance if the goods are independent in consumption, then the vertical restraint on the monopoly good is also set below the monopoly level. In both cases, antitrust regulations that prohibit below-cost pricing are pro-competitive, lowering both retail prices.

Antitrust law also limits vertical restraints directly (Comanor and Rey, 1997). Is a flexible antitrust approach to vertical restraints, as ruled by the U.S. Supreme Court in *Leegan Creative Leather Products, Inc. v. PSKS, Inc.* (Slip Op. No. 06–480, 2007), in the public interest? In our analysis, the vertical restraint enables contracting parties to support the integrated monopoly outcome. However, welfare implications of the practice depend upon the baseline selected for comparison. If no contracts are allowed at all, so that all products are supplied by wholesale pricing, double-marginalization of the monopoly good spurs a retail price that is higher than its monopoly level. Vertical contracts can have favorable welfare properties in this case. If two-part contracts are allowed, then our analysis indicates that *also* allowing a minimum RPM restraint reduces welfare; however, maximum RPM can produce pro-competitive effects, even when the design of the vertical restraint is to achieve horizontal control of the marketplace. Broadly speaking, this logic supports a flexible "rule of reason" for judging the legality of vertical restraints, but also argues for careful scrutiny of the impact of vertical restraints on cross-market competition.

Appendix

Proof of Proposition 1. Properties (*i*) and (*ii*) follow from eq. (11). For equilibrium retail profit per-customer, we turn to the first order condition for a retailer's choice of p^2:

(A1) $F_1(w^1, c^2) = -\Pi(p^{1*}, p^{2*}; w^1, c^2)\, y^2(p^{1*}, p^{2*}) - t\,(w^1 - c^1)\, \partial y^1(p^{1*}, p^{2*})/\partial p^2 = 0.$

$$\Rightarrow \qquad \Pi() = -t\,(w^1 - c^1)\,(\partial y^1()/\partial p^2)/\,y^2 .$$

The claimed signs follow from (A1) and properties (i)-(ii). QED.

Proof of Corollary 1. Substituting (6) into (5) (evaluated at (p^{1*}, p^{2*})) gives:

$$\partial\pi_1/\partial p^1 = (2y^2)^{-1}\,\{(w^1 - c^1)\,A^1 - (w^2 - c^2)\,A^2\}$$

where $A^i \equiv y^i(\partial y^i()/\partial p^j) - y^j(\partial y^i()/\partial p^i)$, $j \neq i$, and $A^i > 0$ for $i \in \{1,2\}$ by our initial assumption that $|\,d\ln u_i / d\ln y^i\,| > |\,d\ln u_i / d\ln y^j\,|$ for $j \neq i$. Hence, at $p^1 = p^{1*}$, with $w^2 = c^2$, $\partial\pi_1/\partial p^1 < 0$ for strong substitutes and $\partial\pi_1/\partial p^1 > 0$ otherwise. QED.

Proof of Corollary 2. *Preliminaries*. Define $(p^1(w), p^2(w))$ that solve equations (5)-(6) in the symmetric equilibrium (where $w = w^1$ and $w^2 = c^2$). Further define the wholesale prices that solve equations (8) and (10) respectively,

$$w_{(8)}:\; p^1(w) = p^{1*}, \quad w_{(10)}:\; p^2(w) = p^{2*},$$

where $w_{(8)} > c^1$ (from eq. (8)) and $w_{(10)} <$ (>) c^1 for strong substitutes (weak substitutes, independent goods and complements) (from eq. (10)). Note from eq. (9), in the symmetric equilibrium,

(A2a) $\qquad p^2(w_{(8)}) < p^{2*}.$

Similarly, evaluating eq. (5) with $w^1 = w_{(10)}$, we have

$$\partial\pi_1(p^{1*}, p^{2*}; \bar{u}_2, w^1, c^2)/\partial p_1^1 =$$

$$\varphi\Pi^*\{(\partial\varphi/\partial p_1^1)(\partial y^1/\partial p_1^2) - (\partial\varphi/\partial p_2^2)(\partial y^1/\partial p_1^1)\}/\{\varphi(\partial y^1/\partial p_1^2) + y^1(\partial\varphi/\partial p_1^2)\}.$$

Hence, in a symmetric equilibrium,

(A2b) $\qquad p^1(w_{(10)}) <(>)\; p^{1*}$ for strong substitutes (other goods).

Constrained to two-part contracts, the monopoly manufacturer's choice problem is:

(A3) $\max_w \Pi(p^1(w), p^2(w); c^1, c^2)$.

Assumption. For a relevant range of w, (A1) $p^1(w)$ and $p^2(w)$ are monotone, and (A2) $\partial^2\Pi/\partial p^1\partial p^2 \overset{s}{=} \partial y^1/\partial p^2$ at $(p^1(w), p^2(w))$, where

$$\partial^2\Pi/\partial p^1\partial p^2 = 2\,(\partial y^1/\partial p^2) + \sum_{i=1,2} (p^i\text{-}c^i)(\,\partial^2 y^i/\partial p^1\partial p^2).$$

Assumption A1 ensures that the $p^i(w)$ functions are well-behaved. Assumption A2, like the assumption of concavity of Π (note 6), is necessary to avoid dominance of third-order effects, and is satisfied if third order derivatives of u are sufficiently small relative to second order derivatives.

By Assumption A1, we can define the inverse function,

(A4a) $w^1(p^1)$: $p^1(\,w^1(p^1)) = p^1$,

and the good 2 retail price mapping,

(A4b) $\underline{p}^2(\,p^1) = p^2(\,w^1(p^1))$.

(A4) implies the equivalent monopoly choice problem,

(A3') $\max_{p1} \Pi(p^1, \underline{p}^2(p^1); c^1, c^2)$.

(I) *Case of Strong Substitutes*. Observe:

Observation 1. By (A2) and (A4), we have the two points on the $\underline{p}^2(\,p^1)$ function,

$$(p^{1*}, \underline{p}^2(\,p^{1*}) = p^2(w_{(8)}) < p^{2*}) \quad \text{and} \quad (p^1(w_{(10)}) < p^{1*}, p^{2*}).$$

Hence, by Assumption A1,

(A5) $\mathrm{d}\,\underline{p}^2(\,p^1)/\mathrm{d}p^1 < 0$.

Observation 2. For $p^1 > p^{1*}$, we have (by (A5)) $\underline{p}^2(\,p^1) < p^2(w_{(8)}) < p^{2*}$ and hence,

(A6) $\Pi(p^1, \underline{p}^2(\,p^1); c^1, c^2) = \Pi(p^{1*}, \underline{p}^2(w_{(8)}); c^1, c^2) - \displaystyle\int_{p^2(p^1)}^{p^2(w_{(8)})} [\partial\Pi(p^{1*}, p^2;.))/\partial p^2]\, \mathrm{d}p^2$

$$+ \int_{p^{1*}}^{p^{1}} \{[\partial\Pi(p^1, p^{2*};.))/\partial p^1] - \int_{\underline{p}^{2}}^{p^{2*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^2\}\, dp^1 < \Pi(p^{1*}, p^2(w_{(8)}); c^1, c^2).$$

The inequality is due to the definition of (p^{1*}, p^{2*}), concavity of Π, $p^2(w_{(8)}) < p^{2*}$, and $[\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1] > 0$ for strong substitutes (Assumption A2).

Similarly, for $p^1 < p^1(w_{(10)}) < p^{1*}$, we have (by (A5)) $\underline{p}^2(p^1) > p^{2*}$ and hence,

(A7) $$\Pi(p^1, \underline{p}^2(p^1); c^1, c^2) = \Pi(p^1(w_{(10)}), \underline{p}^{2*}; c^1, c^2) - \int_{p^1}^{p^1(w_{(10)})} [\partial\Pi(p^1, p^{2*};.))/\partial p^1]\, dp^1$$

$$+ \int_{p^{2*}}^{\underline{p}^2(p^1)} \{[\partial\Pi(p^1, p^{2*};.))/\partial p^2] - \int_{p^1}^{p^{1*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^1\} dp^2 < \Pi(p^1(w_{(10)}), \underline{p}^{2*}; c^1, c^2).$$

By (A6)-(A7), any solution to (A3') is a $p^1 \in [p^1(w_{(10)}), p^{1*}]$.

Observation 3. Differentiating (A3') and evaluating at $p^1 = p^1(w_{(10)})$, where $\underline{p}^2(p^1) = p^{2*}$,

(A8) $$d\,\Pi(p^1(w_{(10)}), \underline{p}^2(p^1(w_{(10)})); c^1, c^2) / d\,p^1 = [\partial\Pi()/\partial p^1] + [\partial\Pi()/\partial p^2][d\underline{p}^2/dp^1] > 0,$$

where the inequality follows from: (i) $p^1(w_{(10)}) < p^{1*}$, implying

$$[\partial\Pi()/\partial p^1] = [\partial\Pi(p^1, p^{2*};.)/\partial p^1] > 0;$$

(ii) $d\underline{p}^2(p^1)/dp^1 < 0$ by (A5), and (iii)

$$[\partial\Pi()/\partial p^2] = [\partial\Pi(p^{1*}, p^{2*};.)/\partial p^2] - \int_{p^1(w_{(10)})}^{p^{1*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^1$$

$$= - \int_{p^1(w_{(10)})}^{p^{1*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^1 < 0.$$

Finally, differentiating (A3') at $p^1 = p^{1*}$, where $\underline{p}^2(p^1) = p^2(w_{(8)}) < p^{2*}$,

(A9) $$d\,\Pi(p^{1*}, \underline{p}^2(p^{1*}); c^1, c^2) / d\,p^1 < 0,$$

where the inequality follows from (i)

$$[\partial\Pi()/\partial p^1] = [\partial\Pi(p^{1*}, p^{2*};.)/\partial p^1] - \int_{p^2(w_{(8)})}^{p^{2*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^2$$

$$= - \int_{p^2(w_{(8)})}^{p^{2*}} [\partial^2\Pi(p^1, p^2;.))/\partial p^2\partial p^1]\, dp^2 < 0.$$

(ii) $d\underline{p}^2(p^1)/dp^1 < 0$ by (A5), and (iii) with $p^2(w_{(8)})<p^{2*}$, $[\partial\Pi()/\partial p^2]=[\partial\Pi(p^{1*}, p^2;.)/\partial p^2]>0$.

Together, Observations 1-3 (equations (A5)-(A9)) imply that any solution to (A3') is a $p^1 \in (p^1(w_{(10)}), p^{1*})$, implying that $p^2 \in (p^2(w_{(8)}), p^{2*})$; hence, $p^1 < p^{1*}$ and $p^2 < p^{2*}$.

(II) *Other Cases*. It suffices to consider independent goods. By continuity, the same conclusions will apply to sufficiently weak substitutes and sufficiently weak complements. Here we have two points on the $\underline{p}^2(p^1)$ schedule:

$$(p^{1*}, p^2(w_{(8)}) < p^{2*}) \quad \text{and} \quad (p^1(w_{(10)}) > p^{1*}, p^{2*}).$$

Hence, by Assumption A1, $d\underline{p}^2(p^1)/dp^1 > 0$. Following mathematics similar to those in (A6)-(A7), we have, for $p^1 > p^1(w_{(10)})$ (and hence $p^2 > p^{2*}$),

(A10a) $$\Pi(p^1, \underline{p}^2(p^1); c^1, c^2) < \Pi(p^1(w_{(10)}), \underline{p}^{2*}; c^1, c^2),$$

and, for $p^1 > p^{1*}$ (and hence, $\underline{p}^2(p^1) < p^2(w_{(8)}) < p^{2*}$),

(A10b) $$\Pi(p^1, \underline{p}^2(p^1); c^1, c^2) < \Pi(p^{1*}, \underline{p}^2(w_{(8)}); c^1, c^2).$$

Finally, differentiating (A3') at $p^1=p^{1*}$ (where $\underline{p}^2(p^1)=p^2(w_{(8)})< p^{2*}$),

(A11a) $$d\,\Pi(p^{1*}, \underline{p}^2(p^{1*}); c^1, c^2) / d\,p^1 = [\partial\Pi()/\partial p^2][d\underline{p}^2/dp^1] > 0,$$

where the equality is due to $[\partial\Pi()/\partial p^1] = 0$ at $p^1=p^{1*}$(by the definition of p^{1*}, and the assumption of independent goods), and the inequality follows from $d\underline{p}^2(p^1)/dp^1 > 0$ and, with $p^2(w_{(8)})< p^{2*}$ (and the assumed concavity in Π)

$$[\partial\Pi()/\partial p^2] = [\partial\Pi(p^{1*}, p^2;.)/\partial p^2] > 0.$$

Similarly, at $p^1=p^1(w_{(10)})$, where $\underline{p}^2(p^1)=p^{2*}$, (A11b) $\quad d\,\Pi(p^1(w_{(10)}), \underline{p}^2(p^1(w_{(10)})); c^1, c^2) / dp^1 = [\partial\Pi()/\partial p^1] < 0.$

Together, (A10)-(A11) imply that any solution to (A3') is a $p^1 \in (p^{1*}, p^1(w_{(10)}) > p^{1*})$, implying that $p^2 \in (p^2(w_{(8)}) < p^{2*}, p^{2*})$; hence, $p^1 > p^{1*}$ and $p^2 < p^{2*}$. QED.

Proof of Proposition 3. First note the following (given Assumption 1):

Claim 1. $\partial p^2(w^1, w_1^2; p_2^2)/\partial p_2^2 \overset{s}{=} \Pi(p^{1*}, p_1^2; w^1, w_1^2)$, where "$\overset{s}{=}$" denotes "equals in sign."

Proof of Claim 1. Differentiating the first order condition (FOC) associated with problem (13) at $p^2 = p^2(w^1, w_1^2; p_2^2)$ and making use of the second order condition gives:

$$\partial p^2(\,)/\,\partial p_2^2 \overset{s}{=} (\partial\Pi()/\partial p_1^2)\,[\partial\phi()/\partial\bar{u}][\partial u^*(p^{1*}, p_2^2)/\,\partial p_2^2]$$

$$= \Pi()\{y^2(p^{1*}, p_1^2)/(2t\phi)\}\{y^2(p^{1*}, p_2^2)/(2t)\} \overset{s}{=} \Pi(),$$

where the terms after the equality are derived by substituting from the FOC and expanding the relevant partial derivatives. QED Claim 1.

Claim 2. At $w^2 = c^2$, $F_2(w^1(w^2), w^2) > 0$ in (14) (where $w^1(w^2)$ solves (16)).

Proof of Claim 2. First note that, if $\Pi=0$ and $w^2 = c^2$, then (16) implies that $w^1 = c^1$ and, hence, $\Pi>0$, a contradiction. Therefore, at $(w^1,w^2)=(w^1(c^2),c^2)$, $\Pi\neq 0$. With $\Pi\neq 0$, Claim 1 implies that the first set of right-hand terms in (17) is positive; with $w^2 = c^2$, the second set of right-hand terms in (17) is zero. QED Claim 2.

Claim 3. There is a bounded $\hat{w}^2 > c^2$ such that $F_2(w^1(\hat{w}^2), \hat{w}^2) < 0$ in (17).

Proof of Claim 3. Define $\hat{w}^2$ by $w^1(\hat{w}^2) = c^1$; that is, from (16),

(A12) $\quad \hat{w}^2 - c^2 = \Pi(p^{1*}, p^{2*}; c^1, c^2)\{y^2(p^{1*},p^{2*})/t\} \,/\, \{[y^2(p^{1*}, p^{2*})^2/t]-[\partial y^2(p^{1*},p^{2*})/\partial p^2]\} > 0.$

Also from (16), we have

(A13) $\quad \Pi(p^{1*}, p^{2*}; c^1, \hat{w}^2) \overset{s}{=} \hat{w}^2 - c^2 > 0.$

Hence, by Claim 1 and Assumption 1, $0 < \partial p^2(\,)/\partial p_2^2 < 1$ at $(w^1,w^2) = (c^1, \hat{w}^2)$, which implies (together with $\Pi>0$ and $\hat{w}^2 > c^2$):

(A14) $\quad F_2(c^1, \hat{w}^2) < \Pi()\, y^2() + (\hat{w}^2 - c^2)\, t\, [\partial y^2()/\partial p^2] = -(w^1 - c^1)t(\partial y^1()/\partial p^2) = 0,$

where the first inequality evaluates the right-hand-side of (17) at $\partial p^2(\,)/\partial p_2^2 = 1$; the first equality substitutes from (16); and the final equality is due to $w^1(\hat{w}^2) = c^1$. QED Claim 3.

Claim 4 (Proposition 3). There is a $w^{2*} \in (c^2, \hat{w}^2)$: $(w^1, w^2) = (w^1(w^{2*}), w^{2*})$ solve (16) and (17).

Proof of Claim 4. Follows directly from Claim 2, Claim 3, continuity of $F_2(w^1(w^2), w^2)$ in w^2, and the Intermediate Value Theorem (IVT). QED.

Proof of Proposition 4. First note:

Claim 5. w^{1*} is above or below c^1, depending upon whether Π^{**} is positive or negative, and whether the two goods are complements ($\partial y^1(p^{1*}, p^{2*})/\partial p^2 < 0$) or substitutes ($\partial y^1()/\partial p^2 > 0$) as follows:

	Retail Goods Are	
	Complements	Substitutes
$\Pi^{**}>0$	$w^{1*}>c^1$	$w^{1*}<c^1$
$\Pi^{**}<0$	$w^{1*}<c^1$	$w^{1*}>c^1$

Proof of Claim 5. Substitute (16) into (17), giving us the following necessary condition for (w^{1*}, w^{2*}) to support the integrated optimum:

(A15) $\quad (w^1 - c^1)(\partial y^1(p^{1*}, p^{2*})/\partial p^2)\,(\partial p^2()/\partial p_2^2)$

$$= (w^2 - c^2)\,(1 - (\partial p^2()/\partial p_2^2))[(\partial y^2(p^{1*}, p^{2*})/\partial p^2) - (y^2(p^{1*}, p^{2*})/t)].$$

By Assumption 1 ($\partial p^2()/\partial p_2^2 < 1$) and Proposition 3 ($w^{2*} > c^2$), the term on the right-hand side of (A15) is negative at the optimum. Hence, the term on the left-hand side of (A15) must be negative. Making use of Claim 1, this requirement yields Claim 5. QED Claim 5.

Proposition 4(i) follows from Proposition 1 (Π>0 at w^2=c^2), dΠ(.;w^1(w^2),w^2)/dw^2 > 0 (by δ>0 for complements), and w^{2*}>c^2 (Proposition 3), which together imply Π**>0 and hence (by Claim 5), w^{1*}>c^1. For part (ii), note:

Claim 6. If the goods are weak (strong) substitutes, $\Pi(p^{1*},p^{2*};w^1(c^2),c^2) < (>) \ 0$.

Proof of Claim 6. At $(w^1,w^2)=(c^1,c^2)$, $\Pi(.;c^1,c^2)>0$ and $F_1(c^1,c^2)<0$ (from (16)); moreover, $\partial F_1(w^1,c^2)/\partial w^1 = \delta > (<) \ 0$ (for weak (strong) substitutes); hence, $F_1(w^1,c^2)<0$ for all $w^1 \leq (\geq) \ c^1$ and, in order to satisfy (16), $w^1(c^2) > (<) \ c^1$. With $[\partial y^1()/\partial p^2] > 0$ (substitutes) and $w^1>(<)c^1$, satisfaction of (16) requires that $\Pi(.;w^1(c^2),c^2)$ be negative (positive). QED Claim 6.

From (A13), we have that $\Pi(.;w^1(\hat{w}^2),\hat{w}^2)>0$ for $\hat{w}^2>c^2$. With $\Pi(.;w^1(c^2),c^2)<0$ (Claim 6 for weak substitutes) and $\Pi(.;w^1(\hat{w}^2),\hat{w}^2)>0$, there is a $\hat{w}^{2+} \in (c^2, \hat{w}^2)$: $\Pi(.;w^1(\hat{w}^{2+}),\hat{w}^{2+})=0$ (by continuity of $\Pi(.;w^1(w^2),w^2)$ in w^2 and the IVT). Moreover, at $w^2=\hat{w}^{2+}$, $F_2(w^1(w^2),w^2)<0$ (because $\Pi()=0$ and $w^2=\hat{w}^{2+}>c^2$); hence, given Claim 2, continuity of $F_2(w^1(w^2),w^2)$ in w^2, and the IVT, $w^{2*} \in (c^2,\hat{w}^{2+})$ and $w^{1*}=w^1(w^{2*})$ solve (16) and (17). With $\Pi(.;w^1(\hat{w}^{2+}),\hat{w}^{2+})=0$, $w^{2*} < \hat{w}^{2+}$, and $d\Pi(.;w^1(w^2),w^2)/dw^2>0$ (by δ>0 for weak substitutes), we have Π**<0 and hence (by Claim 5), $w^{1*}> c^1$.

For part (iii), $\Pi^{**}=\Pi(.;w^1(w^{2*}),w^{2*})>0$ follows from: (a) $\Pi(.;w^1(c^2),c^2)>0$ (Claim 6 for strong substitutes); (b) $\Pi(.;w^1(\hat{w}^2),\hat{w}^2)>0$ (from (A12)-(A13)); (c) $w^{2*} \in (c^2, \hat{w}^2)$ (Claim 4 of Proposition 3) ; and (d) $d\Pi(.;w^1(w^2),w^2)/dw^2 \leq 0$ (by $\delta \leq 0$ for strong substitutes). Hence, $w^{1*}< c^1$ follows from Claim 5. QED.

Proof of Corollary 3. The Corollary follows directly from Propositions 2-3, and equation (18).

Proof of Proposition 5. With $\hat{w}^2$ as defined in (16'), observe that $F_2(c^1,\hat{w}^2) < 0$ (Claim 3). Now consider $w^1< c^1$. Substituting for $(\hat{w}^2 - c^2)\gamma$ in F_2 of equation (17):

(A16) $F_2(w^1, \hat{w}^2) = [(\partial p^2/\partial p_2^2)\, \Pi(p^{1*}, p^{2*}; w^1, c^2) - \Pi^*]\, y^2.$

Define $\omega = -w^1$ and expand the first right hand term in (A16):

(A17) $(\partial p^2/\partial p_2^2)\, \Pi(p^{1*}, p^{2*}; w^1, c^2) = A(\omega)/B(\omega),$

where $A(\omega) = (y^2)^2\, \Pi(.;-\omega = w^1, w^2)\, \Pi(.; -\omega = w^1, c^2)$, and

$B(\omega) = \Pi(\cdot; -\omega = w^1, w^2)\, [2(y^2)^2 + t(\partial y^2/\partial p^2)\,] - t^2[2\,(\partial y^2/\partial p^2) + \sum_{i=1,2} (p^i - w^i)(\, \partial^2 y^i/\partial (p^2)^2)],$

all evaluated at $(p^{1*}, p^{2*}, \hat{w}^2)$. Taking derivatives:

(A18a) $\partial A/\partial \omega = (y^2)^2\, [\Pi(.;-\omega = w^1, w^2) + \Pi(.; -\omega = w^1, c^2)]\, y^1 > 0,$

(A18b) $\partial B/\partial \omega = y^1\, [2(y^2)^2 + t(\partial y^2/\partial p^2)\,] + t^2[\partial^2 y^1/\partial (p^2)^2] \equiv z,$

where the inequality in (A18a) follows from $\Pi(.; -\omega = w^1, c^2) > 0$ (with $w^1 \leq c^1$ and $\Pi^* > 0$) and $\Pi(.; -\omega = w^1, w^2) > 0$ (from eq. (16), $w^2 = \hat{w}^2 > c^2$ (eq. (16'')), $w^1 \leq c^1$, $(\partial y^2/\partial p^2) < 0$, and with $\delta = 0$, $\partial y^1/\partial p^2 > 0$). Note that $\partial B/\partial \omega$ in (A18b) is a constant (invariant to ω) z.

Claim 7. There is a $w^{1*} \in (-\infty, c^1)$: $F_2(w^{1*}, \hat{w}^2) > 0$.

Proof of Claim 7. Define $\omega_0 = -c^1$. By Claim 3 and (A16), $A(\omega_0)/B(\omega_0) < \Pi^*$. There are three cases: (i) $z < 0$. Let $\Delta \equiv .B(\omega_0)/z > 0$, where the inequality is due to $B(\omega_0) > 0$ (by second order conditions for the retailer's choice of p^2) and $z < 0$. Consider $\omega_1 = \omega_0 + \Delta$. By construction, $B(\omega_1) = 0$ and $B(\omega) > 0$ for $\omega \in (\omega_0, \omega_1)$. Hence, $\lim_{\omega \to \omega_1} (A/B) = \infty$, and by the IVT, there is a $\omega^* \in (\omega_0, \omega_1)$: $(A/B) > \Pi^*$. (ii) $z = 0$. With $\lim_{\omega \to \infty} A(\omega) = \infty$ and $B(\omega) = B(\omega_0) > 0$, there is a $\omega^* \in (\omega_0, \infty)$: $(A/B) > \Pi^*$. (iii) $z > 0$. Now we have $\lim_{\omega \to \infty} A(\omega) = \lim_{\omega \to \infty} B(\omega) = \infty$. By L'Hopital's rule, $\lim_{\omega \to \infty} [A(\omega)/B(\omega)] = \lim_{\omega \to \infty} [\partial A/\partial \omega]/z = \infty$. Hence, for all cases, there is a $\omega^* \in (\omega_0, \infty)$: $(A/B) > \Pi^*$. The Claim now follows from the definition of ω and equations (A16)-(A17). QED Claim 7.

The Proposition follows directly from Claims 3 and 7 and the IVT. QED.

Proof of Proposition 6. The claimed equilibrium properties of w and Π^{***} follow directly from equations (23)-(24). By second order conditions and our premise that $\partial p^{1A+}/\partial p^{1B}<1$ (note 27), we have (recalling notes 26 and 27),

(A19) $$dp^{1**}/dw \overset{s}{=} \partial p^{1A+}/\partial w \overset{s}{=} (\partial y^{1B}/\partial p^{1A})\, \Pi^{**}_{2,2} - (\partial y^{1B}/\partial p^{2})\, \Pi^{**}_{1A,2},$$

(A20) $$dp^{2**}/dw = (\partial p^{2+}/\partial w) + (\partial p^{2+}/\partial p^{1B})\, (dp^{1**}/dw),$$

where

(A21) $$\partial p^{2+}/\partial w \overset{s}{=} (\partial y^{1A}/\partial p^{2})\, \Pi^{**}_{1A,1A} - (\partial y^{1B}/\partial p^{1A})\, \Pi^{**}_{1A,2}$$

(A22) $$(\partial p^{2+}/\partial p^{1B}) \overset{s}{=} \Pi^{**}_{1A,2}\, \Pi^{**}_{1A,1B} - \Pi^{**}_{1B,2}\, \Pi^{**}_{1A,1A}\ .$$

For independent goods ($\partial y^{1i}/\partial p^{2}=0$, $i\in\{A,B\}$), the right hand side of (A19) is negative (with $\partial y^{1B}/\partial p^{2}=0$, $\partial y^{1B}/\partial p^{1A}>0$ and $\Pi^{**}_{2,2}<0$ by second order conditions) and the right hand side of (A20) equals zero (with $\partial y^{1B}/\partial p^{2}=0$, and $\Pi^{**}_{1A,2}=\Pi^{**}_{1B,2}=0$); hence, with $w>c$ in equilibrium, $p^{1**}(w)< p^{1**}(c)= p^{1*}$ and $p^{2**}(w)< p^{2**}(c)= p^{2*}$. For other cases, we assume that the conditions described in note 26 are satisfied, so that $dp^{1**}(w)/dw<0$. In addition, we assume that $\Pi^{**}_{i,j} \overset{s}{=} \partial y^{i}/\partial p^{j}=0$ for $j\neq i$, which will be true if third order derivatives of u are sufficiently small. Hence, appealing to second order conditions, $dp^{2**}(w)/dw<0$ for substitutes and $dp^{2**}(w)/dw>0$ for complements. Hence, for strong substitutes, we have $w<c$, $p^{1**}(w)> p^{1**}(c)= p^{1*}$ and $p^{2**}(w)> p^{2**}(c)= p^{2*}$; for weak substitutes, we have $w>c$, $p^{1**}(w)< p^{1**}(c)= p^{1*}$ and $p^{2**}(w)< p^{2**}(c)= p^{2*}$; and for complements, we have $w>c$, $p^{1**}(w)< p^{1**}(c)= p^{1*}$ and $p^{2**}(w)> p^{2**}(c)= p^{2*}$. QED.

References

Allain, M.L. and C. Chambolle. 2005. "Loss-Leaders Banning Laws as Vertical Restraints." *Journal of Agricultural & Food Industrial Organization* 3 (5).

Bagwell, K. and G. Ramey. 1994. "Advertising and Coordination." *Review of Economic Studies* 61: 153-72.

Bliss, C. 1988. "A Theory of Retail Pricing." *Journal of Industrial Economics* 36: 375-91.

Bolton, P. and G. Bonanno. 1988. "Vertical Restraints in a Model of Vertical Differentiation." *Quarterly Journal of Economics* 103: 555-70.

Bork, R.H. 1966. "The Rule of Reason and the Per Se Concept." *Yale Law Journal* 75: 373-475.

Carbajo, J., D. de Meza, and D. Seidmann. 1990. "A Strategic Motivation for Commodity Bundling." *Journal of Industrial Economics* 38: 283-97.

Comanor, W. and P. Rey. 1997. "Competition Policy towards Vertical Restraints in Europe and the United States." *Empirica* 24: 37-52.

Crémer, J. and M. H. Riordan. 1987. "On Governing Multilateral Transactions with Bilateral Contracts." *RAND Journal of Economics* 18: 436-51.

Degraba, P. 2006. "The Loss Leader is a Turkey: Targeted Discounts from Multiproduct Competitors." *International Journal of Industrial Organization* 24: 613-28.

Degraba, P. and R. Mohammed. 1999. "Intertemporal Mixed Bundling and Buying Frenzies." *RAND Journal of Economics* 30: 694-718.

Eunson, L. 2002. "The Politics Behind Gas Price Spikes." *Stanford Business* 71: http://www.gsb.stanford.edu/community/bmag/sbsm0211/faculty_research_polecon_gasoline.html

Federal Trade Commission. 2003. *Slotting Allowances in the Retail Grocery Industry*: http://www.ftc.gov/opa/2003/11/slottingallowance.htm.

Innes, R. and S. F. Hamilton. 2006. "Naked Slotting Fees for Vertical Control of Multi-Product Retail Markets." *International Journal of Industrial Organization* 24: 308-18.

Klein, B. and K. Murphy. 1988. "Vertical Restraints as Contract Enforcement Mechanisms." *Journal of Law and Economics* 31: 265-97.

Kühn, K-U. 1997. “Nonlinear Pricing in Vertically Related Duopolies.” *RAND Journal of Economics* 28: 37-62.

Lal, R. and C. Matutes. 1994. “Retail Pricing and Advertising Strategies.” *Journal of Business* 67: 345-70.

Macleod, W.B. and J. Malcomson. 1995. “Contract Bargaining with Symmetric Information.” *Canadian Journal of Economics* 28: 336-67.

Marion, B.W. 1998. “Changing Power Relationships in U.S. Food Industry: Brokerage Arrangements for Private Label Products.” *Agribusiness* 14: 85–93.

Martimort, D. 1996. “Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory.” *RAND Journal of Economics* 27: 1-31.

Marvel, H. and S. McCafferty. 1984. “Resale Price Maintenance and Quality Certification.” *RAND Journal of Economics* 15: 340-59.

Marx, L.M., and G. Shaffer. 1999. “Predatory Accommodation: Below-Cost Pricing without Exclusion in Intermediate Products Markets.” *RAND Journal of Economics* 30: 22-43.

Marx, L.M., and G. Shaffer. 2004. “Upfront Payments and Exclusion in Downstream Markets.” Working Paper, Duke University and University of Rochester.

Mathewson, G. and R. Winter. 1984. “An Economic Theory of Vertical Restraints,” *RAND Journal of Economics* 15: 27-38.

Mathewson, G. and R. Winter. 1997. “Tying as a Response to Demand Uncertainty,” *RAND Journal of Economics* 28: 566-83.

Mathewson, G. and R. Winter. 1998. “The Law and Economics of Resale Price Maintenance,” *Review of Industrial Organization* 13: 57-84.

McAfee, R.P., J. McMillan and M. Whinston. 1989. “Multiproduct Monopoly, Commodity Bundling, and Correlation of Values.” *Quarterly Journal of Economics* 104: 371-84.

Meyer, D. and J. Fischer. 2004. “The Economics of Price Zones and Territorial Restrictions in Gasoline Marketing.” Working Paper, FTC. www.fts.gov/be/workpapers/wp271.pdf.

Nalebuff, B. 2004. “Bundling as an Entry Barrier.” *Quarterly Journal of Economics* 119: 159-87.

O'Brien, D. P. and G. Shaffer. 1992. "Vertical Control with Bilateral Contracts." *RAND Journal of Economics* 23: 299-308.

PLBuyer (July 2004): http://www.privatelabelbuyer.com/content.php?s=PB/2004/07&p=8.

Posner, R.A. 1976. *Antitrust Law: An Economic Perspective*, University of Chicago Press: Chicago.

Reiffen, D. 1999. "On the Equivalence of Resale Price Maintenance and Quantity Restrictions." *International Journal of Industrial Organization* 17: 277-88.

Rey, P. and J. Tirole. 1986. "The Logic of Vertical Restraints." *American Economic Review* 76: 921-39.

Rey, P. and T. Vergé. 2004. "Resale Price Maintenance and Horizontal Cartel." CMPO Working Paper Series No. 02/047.

Rey, P., J. Thal, and T. Vergé. 2006. "Slotting Allowances and Conditional Payments." Working Paper, CREST-LEI and University of Toulouse.

Rubinstein, A. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50: 97-109.

Shaffer, G. 1991a. "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices." *RAND Journal of Economics* 22: 120-135.

Shaffer, G. 1991b. "Capturing Strategic Rent: Full-Line Forcing, Brand Discounts, Aggregate Rebates, and Maximum RPM." *Journal of Industrial Economics* 39: 557-75.

Shaked, A. 1987. "Opting Out: Bazaars Versus 'Hi-Tech' Markets." Theoretical Economics Discussion Paper 87/159, London School of Economics.

Supermarket News. (March 13, 1995). "Masters of the House." pp 39-42.

Telser, L. 1960. "Why Should Manufacturers Want Free Trade?" *Journal of Law and Economics* 3: 86-105.

Whinston, M. 1990. "Tying, Foreclosure and Exclusion." *American Economic Review* 80: 837-59.

Winter, R. 1993. "Vertical Control and Price Versus Nonprice Competition." *Quarterly Journal of Economics* 108: 61-76.

TABLE 1.
Summary of Contract Outcomes

Retail Goods Are:	Wholesale Price	Variable Retail Profit
Complements	$w^{1*} > c^1$	$\Pi > 0$
Independent	$w^{1*} > p^{1*} > c^1$	$\Pi = 0$
Weak Substitutes	$w^{1*} > p^{1*} > c^1$	$\Pi < 0$
Strong Substitutes	$w^{1*} < c^1$	$\Pi > 0$